Propagator of a scalar field on a stationary slowly varying gravitational background

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Abstract

The propagator of a scalar field on a stationary slowly varying in space gravitational background is derived retaining only the second derivatives of the metric. The corresponding one-loop effective action is constructed. The propagator and the effective action turn out to depend nontrivially on the Killing vector defining the vacuum state and the Hamiltonian of a scalar field. The Hawking particle production is described in the quasiclassical approximation and the quasiclassical formula for the Hawking temperature is derived. The behaviour of the Unruh detector on a curved background is considered and the quasiclassical formula for the Unruh acceleration determining the Unruh temperature is derived. The radiation reaction problem on a curved background is discussed in view of the new approximate expression for the propagator. The correction to the mass squared of a scalar particle on a stationary gravitational background is obtained. This correction is analogous to the quantum correction to the particle mass in a strong electromagnetic field. For a vacuum solution to the Einstein equations, it is equal to minus one-fourth of the free fall acceleration squared.

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I. INTRODUCTION

In calculating the the propagator of quantum fields on a fixed curved background, we encounter severe technical problems (see for review, [1]) such that the exact expression can be obtained only for a very limited class of spacetimes possessing a large symmetry group. However, for many physical problems we need an approximate expression, at least, under the assumption that the gravitational field varies slowly. This is the so-called low energy approximation [1, 2]. In the present paper we shall obtain the explicit expression for the propagator of a scalar field on a stationary slowly varying in space gravitational background retaining only the second derivatives of the metric.

The essence of the problem in constructing the approximate expression for the propagator in the external gravitational field is related to the fact that the background field stays at the second derivatives of the Klein-Gordon operator. In other words, this field enters the principal symbol of the operator. So, in the leading approximation when the quadratic part of the symbol is taken into account, the metric field should be considered as a constant matrix. Of course, such an approximation is inappropriate for sufficiently large point separation of the propagator or for the problems where the next to leading corrections to the propagator depending on the derivatives of the metric are relevant. The Hawking particle production [3], the Unruh effect [4], and the radiation reaction problem [5–8] on a curved background are among such physical problems. We shall derive the next to leading correction to the propagator employing appreciably the stationarity of the background metric.

It was shown in [9] (see also [10, 11]) that for stationary backgrounds the Killing vector ξ^μ appears in the effective action in a nontrivial way, that is the expressions involving it cannot be rewritten in terms of the local expressions involving the metric and the curvature alone. This fact can be used to measure experimentally the variation of the Killing vector (up to multiplication by a constant) from point to point. The present paper continues the research in this direction. We are about to investigate in detail how the Killing vector field influences (determines) the dynamics of the matter fields. The first object for such an investigation is, of course, the propagator of a quantum field. Making use of the explicit expression for the propagator, we shall derive the analog of the Heisenberg-Euler [12, 13] effective action which, as we shall see, depends nontrivially on the vector field ξ^μ. We shall also obtain the quasiclassical formulas for the Hawking particle production [3] and for the Unruh temperature [4] of a detector moving in a stationary gravitational field. The quantum correction to the mass of a particle will be also found. This correction is not due to the Higgs mechanism, but a direct one, and analogous to the correction to the mass of a

charged particle in a strong electromagnetic field [14]. On the vacuum solutions to the Einstein equations ($R_{\mu\nu} = 0$), the correction to the mass squared is found to be equal to minus one-fourth of the free fall acceleration squared. The radiation reaction effect on a curved background as a mean to measure the direction of the Killing vector will be also discussed. Of course, one may contrive less sophisticated methods to measure the various contractions of the Killing vector [15–18]. We address just these problems as their solutions follow immediately from the form of the propagator. The mention should be made that the above two problems concerning the Unruh effect and the radiation reaction effect are widely discussed in the literature arriving at the contradictory conclusions (see, e.g., [19, 20]). One of the aim of the present paper is to provide a more solid mathematical background for these considerations on a curved background.

The paper is organized as follows. In Sec. II, the derivative expansion of the Feynman propagator is considered near its diagonal up to the second derivatives of the stationary metric field. The stationarity of the metric is essentially employed and the generalized Schwinger-DeWitt technique [21] adapted to the $(3+1)$-decomposition is used. We shall also see that the simple derivative expansion has a large degree of ambiguity and cannot be used to restore the propagator unambiguously. In Sec. III, the derivative expansion is resummed making use of the Schwinger approach [13], and the low energy expansion [1, 2] of the propagator is constructed thereby. Here we shall derive the explicit expression for the propagator (positive-frequency function) as the double integral over the frequencies of the modes and an auxiliary variable – the Fock proper-time [22]. Section IV is devoted to a further simplification of the expression for the propagator under the reasonable additional assumptions on the fields and the point separation. Several physical implications following immediately from the form of the propagator will be discussed. We shall obtain the spectral representation of the propagator and its representation in terms of the proper-time. The expression for the one-loop correction to the effective action will be also derived. In conclusion, we shall discuss some implications of the results of the present paper. In appendix A, several formulas concerning the calculus on the spacetime with the Killing vector are presented. There, in particular, the expressions for the 4-vectors of the gravitational and inertial forces constructed with the help of the Killing vector field ξ^μ will be given. Some useful expansions needed in the course of the derivation of the approximate expression for the propagator are collected in appendix B.

We shall use the conventions adopted in [21, 23]

$$R^{\alpha}{}_{\beta\mu\nu} = \partial_{[\mu}\Gamma^{\alpha}_{\nu]\beta} + \Gamma^{\alpha}_{[\mu\gamma}\Gamma^{\gamma}_{\nu]\beta}, \qquad R_{\mu\nu} = R^{\alpha}{}_{\mu\alpha\nu}, \qquad R = R^{\mu}_{\mu}, \tag{1}$$

for the curvatures and other structures appearing in the heat kernel expansion. The square and

round brackets at a pair of indices denote antisymmetrization and symmetrization without 1/2, respectively. The Greek indices are raised and lowered by the metric $g_{\mu\nu}$ which has the signature +2. Also we assume that the metric possesses the timelike Killing vector ξ^μ

$$\mathcal{L}_\xi g_{\mu\nu} = 0, \qquad \xi^2 = g_{\mu\nu}\xi^\mu\xi^\nu < 0, \tag{2}$$

that allows us to make the decomposition [24]

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu =: \xi^2(g_\mu dx^\mu)^2 + \bar{g}_{\mu\nu}dx^\mu dx^\nu, \tag{3}$$

where $g_\mu = \xi_\mu/\xi^2$ is a one-form dual to the Killing vector (the Tolman temperature one-form). In the system of coordinates, where $\xi^\mu = (1,0,0,0)$, we have the relations

$$\begin{gathered}\bar{g}_{ik}g^{kj} = \delta_i^j, \qquad \xi^2 \det \bar{g}_{ij} = g, \qquad g^{00} - \bar{g}_{ij}g^{0i}g^{0j} = (g_{00})^{-1},\\ \bar{g}_{\mu\nu} = \begin{bmatrix} 0 & 0 \\ 0 & g_{ij} - \frac{g_{i0}g_{j0}}{g_{00}} \end{bmatrix}, \qquad \bar{g}^{\mu\nu} = g^{\mu\nu} - \xi^2 g^\mu g^\nu = \begin{bmatrix} g^{00} - g_{00}^{-1} & g^{0j} \\ g^{i0} & g^{ij} \end{bmatrix}.\end{gathered} \tag{4}$$

The the Latin indices corresponding to the space are raised and lowered by the positive-definite metric $\bar{g}_{ij}$. The curvatures associated with this metric will be distinguished by the overbars, e.g., $\bar{R}$. The system of units is chosen such that $c = \hbar = 1$.

II. ASYMPTOTIC EXPANSION OF THE PROPAGATOR NEAR THE DIAGONAL

Our aim in this section is to obtain the derivative expansion of the Feynman propagator near its diagonal up to the second derivatives of the stationary metric field $g_{\mu\nu}$. We shall take exactly into account the symmetry of the background metric with respect to translations in time and employ the so-called generalized Schwinger-DeWitt technique [21] adapted to the $(3+1)$-decomposition. Hereinafter we shall work in the reference frame where the Killing vector is straighten ($\xi^\mu = (1,0,0,0)$). Besides all the formulas presented in this section are valid for an arbitrary spacetime dimension $(d+1)$.

Let

$$\begin{aligned}H(x,y) = (\nabla_x^2 - m^2)\frac{\delta(x-y)}{|g|^{1/4}(x)|g|^{1/4}(y)} =&\\ = |g|^{-1/4}(x)\Big[|g|^{-1/4}(x)\partial_\mu\sqrt{|g|}g^{\mu\nu}\partial_\nu|g|^{-1/4}(x) - m^2\Big]\frac{\delta(x-y)}{|g|^{1/4}(y)}\end{aligned} \tag{5}$$

be the bi-scalar kernel of the Klein-Gordon operator. Supposing the metric does not depend on time t, we can write

$$\begin{aligned} H(x,y) &=: |\xi^2|^{-1/4}(\mathbf{x}) \int \frac{d\omega}{2\pi} e^{-i\omega t} H(\omega;\mathbf{x},\mathbf{y}) |\xi^2|^{-1/4}(\mathbf{y}), \\ H(\omega;\mathbf{x},\mathbf{y}) &= \left[-\frac{\omega^2}{\xi^2} + (\bar{\nabla}_i + i\omega g_i)^2 - m^2 - \frac{1}{2}\bar{\nabla}^i h_i - \frac{1}{4} h_i h^i \right] \frac{\delta(\mathbf{x}-\mathbf{y})}{\bar{g}^{1/4}(\mathbf{x})\bar{g}^{1/4}(\mathbf{y})}, \end{aligned} \tag{6}$$

where $x = (t, \mathbf{x})$ and $y = (0, \mathbf{y})$, the Levi-Civita connection $\bar{\nabla}_i$ is constructed in terms of the metric $\bar{g}_{ij}$, and we have introduced the notation $h_\mu := \partial_\mu \ln \sqrt{|\xi^2|}$. Notice that the one-form g_i plays the role of a $U(1)$ gauge field with the charge ω (the energy). The kernel $H(\omega;\mathbf{x},\mathbf{y})$ is a bi-scalar with respect to the spatial diffeomorphisms. Then the positive-frequency function defined with respect to the standard vacuum state for the stationary backgrounds (in black hole physics it is known as the Boulware vacuum [25]) is written as

$$\begin{aligned} D^{(+)}(x,y) &:= -i\langle\phi(x)\phi(y)\rangle = -i \int_0^\infty d\omega e^{-i\omega t_-} |\xi^2|^{-1/4}(\mathbf{x}) \langle\mathbf{x}|\delta(H(\omega))|\mathbf{y}\rangle |\xi^2|^{-1/4}(\mathbf{y}), \\ \langle\mathbf{x}|\delta(H(\omega))|\mathbf{y}\rangle &= \int \frac{ds}{2\pi} \langle\mathbf{x}|e^{isH(\omega)}|\mathbf{y}\rangle =: \int \frac{ds}{2\pi} G(\omega,s;\mathbf{x},\mathbf{y}), \end{aligned} \tag{7}$$

where $t_- := t - i0$ and the integration contour in the s-plane passes a little bit lower than the real axis. The Feynman propagator is expressed in terms of the positive-frequency function: at $t > 0$ the propagator coincides with it, and at $t < 0$ we should make the replacement,

$$t_- \to -t + i0, \qquad \mathbf{x} \leftrightarrow \mathbf{y}, \tag{8}$$

in $D^{(+)}(x,y)$. So, it is sufficient to calculate the positive-frequency function only.

To this end, we can apply the generalized Schwinger-DeWitt technique to the heat kernel $G(\omega,s;\mathbf{x},\mathbf{y})$ assuming that the points $\mathbf{x}$ and $\mathbf{y}$ can be connected by a unique geodesic of the metric $\bar{g}_{ij}$. Making the derivative expansion, it is convenient to use the covariant qp-symbol of the heat kernel [26]

$$\begin{aligned} G = \exp\left\{ is(-\frac{\omega^2}{\xi^2} + D_i^2 - m^2 - \frac{1}{2}\bar{\nabla}^i h_i - \frac{1}{4}h_i h^i) \right\} \int \frac{d^d k'}{(2\pi)^d} \sqrt{\bar{g}'} e^{ik^{i'}\bar{\sigma}_{i'}} a_0(\mathbf{x},\mathbf{y}) = \\ = \int \frac{d^d k'}{(2\pi)^d} \sqrt{\bar{g}'} e^{ik^{i'}\bar{\sigma}_{i'} - is(\frac{\omega^2}{\xi^2} + k^{i'}\bar{\sigma}_{i'i}\bar{\sigma}^i{}_{j'}k^{j'} + m^2 + \frac{1}{2}\bar{\nabla}^i h_i + \frac{1}{4}h_i h^i)} a_0(\mathbf{x},\mathbf{y}) \times \\ \times a_0(\mathbf{y},\mathbf{x}) e^{is(\frac{\omega^2}{\xi^2} + k^{i'}\bar{\sigma}_{i'i}\bar{\sigma}^i{}_{j'}k^{j'} + \frac{1}{2}\bar{\nabla}^i h_i + \frac{1}{4}h_i h^i)} e^{is(-\frac{\omega^2}{\xi^2} + (D_i + ik^{i'}\bar{\sigma}_{i'i})^2 - \frac{1}{2}\bar{\nabla}^i h_i - \frac{1}{4}h_i h^i)} a_0(\mathbf{x},\mathbf{y}), \end{aligned} \tag{9}$$

where the primes denote the quantities and derivatives referring to the point $\mathbf{y}$, and $a_0(\mathbf{x},\mathbf{y})$ is the geodetic parallel displacement operator with respect to the connection $D_i = \bar{\nabla}_i + i\omega g_i$,

$$a_0(\mathbf{x},\mathbf{y}) = e^{-i\omega \int_{\mathbf{y}}^{\mathbf{x}} dx^i g_i}. \tag{10}$$

The integral is taken along the geodesic of the metric $\bar{g}_{ij}$ connecting the points $\mathbf{x}$ and $\mathbf{y}$.

The derivative expansion is obtained expanding the last line in (9) in a covariant Taylor series near the point $\mathbf{x}$. At first, all the exponents in this line are expanded in the series. After that the covariant derivatives D_i^2 are carried through the expression to the right to act on a_0, whereas the derivatives of a_0 are expanded in the covariant Taylor series by the formulas (4.26)-(4.29) in [21]. In the notation of [21]

$$\hat{\mathcal{R}}_{\mu\nu} \to i\omega f_{ij}, \tag{11}$$

where $f_{ij} := \partial_{[i}g_{j]}$. The first exponent in the last line of Eq. (9) provides considerable cancellations in the expansion – all the terms, which do not contain the derivatives of the expression staying in the exponent, are gone away. The exponent in the second line effectively sums such terms (see for the proof [27, 28]). As a result, we obtain the series containing the nonnegative integer powers of $k^{i'}$. Then the Gaussian integral over $k^{i'}$ is performed. In fact, one can replace $k^{i'}$ by $-i\partial/\partial\bar{\sigma}_{i'}$ acting on the result of the Gaussian integration of the expression in the second line in (9)

$$\frac{\bar{\Delta}^{-1}(\mathbf{x},\mathbf{y})}{(4\pi is)^{d/2}} e^{\frac{i}{4s}\bar{\sigma}^{i'}\bar{\sigma}^{-1}_{i'i}\bar{g}^{ij}\bar{\sigma}^{-1}_{jj'}\bar{\sigma}^{j'} - is(m^2+\frac{\omega^2}{\xi^2}+\frac{1}{2}\bar{\nabla}^i h_i + \frac{1}{4}h_i h^i)} a_0(\mathbf{x},\mathbf{y}). \tag{12}$$

After rather lengthy but straightforward calculations one derives

$$\begin{aligned} G = \frac{\bar{\Delta}^{-1}(\mathbf{x},\mathbf{y})}{(4\pi is)^{d/2}} e^{\frac{i}{2s}\bar{\sigma} - is(m^2+\frac{\omega^2}{\xi^2}+\frac{1}{2}\bar{\nabla}^i h_i + \frac{1}{4}h_i h^i)} a_0(\mathbf{x},\mathbf{y}) \Big\{ 1 + \frac{1}{4}\bar{R}_{ij}\bar{\sigma}^i\bar{\sigma}^j + \frac{is}{6}\bar{R} - \\ - \frac{s^2}{2}\Big[\frac{\omega^2}{\xi^2}\Big(\frac{2}{3}\bar{\nabla}^i h_i - \frac{4}{3}h_i h^i\Big) - \frac{\omega}{3s}\bar{\nabla}^j f_{ji}\bar{\sigma}^i - \frac{\omega^2}{2} f_{ij}f^{ij} + \frac{2i}{s}\frac{\omega^2}{\xi^2}h_i\bar{\sigma}^i\Big] - \\ - \frac{is^3}{3}\Big[\frac{\omega^4}{\xi^4}h_i h^i + \frac{\omega^3}{s\xi^2}\bar{\sigma}^i f_{ij}h^j + \frac{2\omega^2}{s^2\xi^2}\Big((h_i\bar{\sigma}^i)^2 - \frac{1}{2}\bar{\nabla}_i h_j \bar{\sigma}^i\bar{\sigma}^j\Big) + \frac{\omega^2}{2is} f_{ij}f^{ij} - \frac{\omega^2}{4s^2}\bar{\sigma}^i f_{ik}f^k{}_j\bar{\sigma}^j\Big] - \frac{s^2\omega^4}{2\xi^4}(h_i\bar{\sigma}^i)^2 \Big\} \end{aligned} \tag{13}$$

up to the second derivatives in the background fields, all the fields being taken at the point $\mathbf{x}$. Notice that in this approach the order of the term in derivatives is counted as the total number of derivatives entering it. For example, the term

$$(h_i h^i)^2 \tag{14}$$

is of the fourth order in derivatives.

The heat kernel $G(\omega, s; \mathbf{x}, \mathbf{y})$ is a unitary operator with respect to the standard measure $\bar{g}^{1/2}$ or, equivalently, it is Hermitian with respect to this measure when s is purely imaginary. It is desirable to preserve this symmetry in the approximate expression for the heat kernel. The expression (13) is not Hermitian at $s = i\tau$ since the points $\mathbf{x}$ and $\mathbf{y}$ enter asymmetrically to it. Therefore, we expand

(13) in the vicinity of the point $p(\mathbf{x},\mathbf{y})$ lying at the middle of the geodesic connecting the points $\mathbf{x}$ and $\mathbf{y}$ (the midpoint prescription) and retain only the terms which are no more than the second order in derivatives of the fields. An inspection of the expansion (13) shows up that we only need to expand ξ^{-2} in the exponent and the last term in the second line, while the fields entering the other terms are simply taken at the midpoint. This gives

$$G=\frac{a_0(\mathbf{x},\mathbf{y})}{(4\pi is)^{d/2}}e^{\frac{i}{2s}\bar\sigma-is(m^2+\frac{\omega^2}{\xi^2}+\frac12\bar\nabla^ih_i+\frac14h_ih^i)}\Big\{1+\frac{1}{12}\bar R_{ij}\bar\sigma^i\bar\sigma^j+\\ +\frac{is}{6}\bar R+\frac{is}{12}\frac{\omega^2}{\xi^2}(\bar\nabla_ih_j-2h_ih_j)\bar\sigma^i\bar\sigma^j-\frac{s^2}{6}\Big[\frac{\omega^2}{\xi^2}\Big(2\bar\nabla^ih_i-4h_ih^i\Big)-\frac{\omega}{s}\bar\nabla^jf_{ji}\bar\sigma^i-\frac{\omega^2}{2}f_{ij}f^{ij}\Big]-\\ -\frac{is^3}{3}\Big[\frac{\omega^4}{\xi^4}h_ih^i+\frac{\omega^3}{s\xi^2}\bar\sigma^if_{ij}h^j-\frac{\omega^2}{4s^2}\bar\sigma^if_{ik}f^k_{\ j}\bar\sigma^j\Big]\Big\},\quad(15)$$

where we also expand the bi-scalar van Vleck determinant $\bar\Delta(\mathbf{x},\mathbf{y})$ (see Appendix B).

Now we should substitute the expansion obtained to the integral (7) and integrate it over ω and s. However, at this point we have a large degree of ambiguity how to represent the substituting expression, and this ambiguity can change substantially the resulting positive-frequency function. For example, we may expand the exponent in (15) and neglect the higher order terms or, conversely, collect some terms in the preexponential factor to the exponent similarly to what we did with the term ω^2/ξ^2. The naive derivative expansion, which we carried out in this section, does not tell us what the right form of the integrand is. We shall address this problem in the next section, while here we establish the relation of the expansion (15) with the standard one [21, 23] written in terms of the geodetic interval of the spacetime rather than $\bar\sigma$. In such a way, we shall make a nontrivial check of the correctness of the expression (15).

In order to obtain the standard asymptotic expansion of the Feynman propagator, it is convenient to employ the Schwinger representation for it

$$D(x,y):=-i\langle T\{\phi(x)\phi(y)\}\rangle=\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}e^{-i\omega t}|\xi^2|^{-1/4}(\mathbf{x})\langle\mathbf{x}|(H(\omega)+i0)^{-1}|\mathbf{y}\rangle|\xi^2|^{-1/4}(\mathbf{y}),\\ \langle\mathbf{x}|(H(\omega)+i0)^{-1}|\mathbf{y}\rangle=\int_0^\infty ids\langle\mathbf{x}|e^{is(H(\omega)+i0)}|\mathbf{y}\rangle=\int_0^\infty idsG(\omega,s;\mathbf{x},\mathbf{y})|_{m^2\to m^2-i0},\quad(16)$$

instead of using Eq. (7). Then we can formally integrate over ω the expansion (15) as the integral of a Gaussian type. This leads to

$$|\xi^2|^{-1/4}G|\xi'^2|^{-1/4}\approx\frac{ie^{\frac{i}{4s}(\xi^2T^2+2\bar\sigma)-ism^2}}{(4\pi is)^{(d+1)/2}}\Big\{\frac{i}{48s}\Big[\xi^2T^2(\bar\nabla_ih_j-2h_ih_j+\xi^2f_{ik}f^k_{\ j})\bar\sigma^i\bar\sigma^j+\\ +2\xi^4T^3\bar\sigma^if_{ij}h^j-\xi^4T^4h^2\Big]+1+\frac{1}{12}\Big[\Big(\bar R_{ij}-h_ih_j-\bar\nabla_ih_j+\frac{\xi^2}{2}f_{ik}f^k_{\ j}\Big)\bar\sigma^i\bar\sigma^j+\xi^2T^2(2h^2-\bar\nabla^ih_i)-\\ -\xi^2T\bar\nabla^jf_{ji}\bar\sigma^i+\frac14\xi^4T^2f^2-3\xi^2T^2h^2+3\xi^2T\bar\sigma^if_{ij}h^j\Big]+\frac{is}{6}\Big[\bar R-2h^2-2\bar\nabla^ih_i-\frac{\xi^2}{4}f^2\Big]\Big\},\quad(17)$$

where $T := t + \int_{\mathbf{y}}^{\mathbf{x}} dx^i g_i$ is a scalar with respect to the general coordinate transformations [24]. Bearing in mind that $\bar{\sigma}_\mu = 0$ at $\mu = 0$ in the adapted system of coordinates (i.e. $\xi^\mu \bar{\sigma}_\mu = 0$) and applying the formulas given in Appendix A, we can write to the same accuracy

$$
\begin{aligned}
|\xi^2|^{-1/4} G |\xi'^2|^{-1/4} &\approx \frac{ie^{\frac{i}{4s}(\xi^2 T^2 + 2\bar{\sigma}) - ism^2}}{(4\pi is)^{(d+1)/2}} \Big\{ -\frac{i\xi^2 T^2}{48s} \big[\xi^2 (T h_\mu + f_{\mu\nu}\bar{\sigma}^\nu)^2 - (\nabla_\mu h_\nu - 2h_\mu h_\nu)\bar{\sigma}^\mu \bar{\sigma}^\nu\big] + \\
&\quad + 1 + \frac{1}{12} R_{\mu\nu} (T\xi^\mu + \bar{\sigma}^\mu)(T\xi^\nu + \bar{\sigma}^\nu) + \frac{is}{6} R \Big\} \approx \\
&\approx \exp\Big[\frac{i}{4s}\big\{\xi^2 T^2 + 2\bar{\sigma} - \frac{\xi^2 T^2}{12}[\xi^2 (T h_\mu + f_{\mu\nu}\bar{\sigma}^\nu)^2 - (\nabla_\mu h_\nu - 2h_\mu h_\nu)\bar{\sigma}^\mu \bar{\sigma}^\nu]\big\} - ism^2\Big] \times \\
&\quad \times \frac{i}{(4\pi is)^{(d+1)/2}} \Big\{1 + \frac{1}{12} R_{\mu\nu}(T\xi^\mu + \bar{\sigma}^\mu)(T\xi^\nu + \bar{\sigma}^\nu) + \frac{is}{6} R\Big\}. \qquad (18)
\end{aligned}
$$

Up to the terms of the second order in derivatives, the quantity staying in the curly brackets in the exponent is the geodetic interval squared, and

$$T\xi_\mu + \bar{\sigma}_\mu \approx \sigma_\mu \qquad (19)$$

to the leading order in derivatives.

Indeed, it is sufficient to consider

$$X^2(x,y) := \xi^2(p) T^2 + 2\bar{\sigma} - \frac{\xi^2 T^2}{12}[\xi^2 (T h_\mu + f_{\mu\nu}\bar{\sigma}^\nu)^2 - (\nabla_\mu h_\nu - 2h_\mu h_\nu)\bar{\sigma}^\mu \bar{\sigma}^\nu] \qquad (20)$$

on the arbitrary worldline $x^\mu(\tau)$, where τ is the natural parameter

$$g_{\mu\nu}(x(\tau)) \dot{x}^\mu(\tau) \dot{x}^\nu(\tau) = -1. \qquad (21)$$

For definiteness, we can take $x^\mu(0) = y = 0$ with zero being the origin of the Riemann normal coordinates of the metric $\bar{g}_{ij}$ (see for details [29]). To fix the frame in the spacetime uniquely, it is convenient to use the Fock gauge [24, 26, 30]

$$g_i(x) x^i = 0 \;\Leftrightarrow\; g_i(x) = \sum_{n=1}^{\infty} \frac{n}{(n+1)!} x^{j_1} \cdots x^{j_n} \partial_{j_1} \ldots \partial_{j_{n-1}} f_{j_n i} = \frac{1}{2}\bar{\sigma}^j f_{ji} + \frac{1}{3}\bar{\sigma}^{j_1}\bar{\sigma}^{j_2}\bar{\nabla}_{j_1} f_{j_2 i} + \ldots, \qquad (22)$$

where x^i are the coordinates of the Riemann frame. This gauge is equivalent to

$$\int_0^{\mathbf{x}} dx^i g_i = 0, \qquad (23)$$

for any x^i of the Riemann normal coordinates, where the integral is taken along a straight line, and so $T = t$. Note that the higher terms of the expansion (22) depend on the curvature [29], but they will be irrelevant in our case as they are of the higher order in derivatives. Then expanding the expression (20) in τ and making use of the relations (21), (22), one arrives at (see Appendix B)

$$X^2 = -\tau^2 - \frac{1}{12}(\nabla_u u)^2_{\tau=0}\tau^4 + \ldots, \qquad u^\mu := \dot{x}^\mu(\tau), \qquad (24)$$

up to the terms of the second order in derivatives of the metric $g_{\mu\nu}$. The Killing vector disappears from the expression. In a free fall, $X^2 = -\tau^2$ and, consequently, with the accuracy we work, X^2 coincides with the geodetic interval squared. Thus we see that the expansion (18) agrees with the standard asymptotic expansion of the heat kernel near the diagonal [21, 23]. So, the expansions for the propagator following from (16) are also the same up to the terms of the second order in derivatives.

III. RESUMMATION OF THE EXPANSION

In the previous section we saw that the resulting expression for the positive-frequency function $D^{(+)}(x,y)$ depends severely on the rearrangements of the derivative expansion. The very fact that we was able to integrate the expansion (15) over the frequency ω is a consequence of the resummation of the derivative expansion, which we have done raising the potential term ω^2/ξ^2 to the exponent. Therefore we need a more reliable procedure to obtain an approximate but adequate expression for the propagator. This procedure should sum an infinite number of terms of the derivative expansion, say, all the terms containing the fields at the given point, their first, second derivatives and not the third derivatives and higher. This is the so-called low energy expansion of the heat kernel [1, 2]. For example, in this case the term of the form (14) must not be neglected. In this section, we obtain the leading term in the low energy expansion of the heat kernel slightly generalizing the standard procedure [13] to the curved spacetime. Now we put $d=3$.

To begin with, we consider a simple model in a flat Euclidean space with the action

$$S[x(\tau)] = \int_0^s d\tau\Big(\frac{1}{2}\dot{x}^2 - A_i(x)\dot{x}^i + \frac{1}{2}E_{ij}x^i x^j\Big), \tag{25}$$

where $A_i = x^j f_{ji}/2$, the field strength matrix f_{ij} is constant and skewsymmetric, E_{ij} is a constant symmetric matrix. This is a general quadratic model and later on we shall see how to use it to construct a reliable positive-frequency function of the scalar field on a curved background. Further, we assume

$$[E,f]=0 \;\Rightarrow\; f = -iH\upsilon^1_{[i}\bar{\upsilon}^1_{j]} =: -iHa_{ij}, \quad E = \lambda_1\upsilon^1_{(i}\bar{\upsilon}^1_{j)} + \lambda_2\upsilon^2_i\upsilon^2_j =: \lambda_1 s_{ij} + \lambda_2\upsilon^2_i\upsilon^2_j. \tag{26}$$

The vectors υ^1_i, $\bar{\upsilon}^1_i$, and υ^2_i are orthonormal with respect to the standard Hermitian scalar product, the overbar denotes complex conjugation, the vector υ^2_i having real components. In this case, the equations of motion for this model can be readily integrated

$$x = \big[(\zeta_1 e^{-i\omega^+\tau} + \zeta_2 e^{-i\omega^-\tau})\upsilon^1 + c.c.\big] + (c_1 e^{\sqrt{\lambda_2}\tau} + c_2 e^{-\sqrt{\lambda_2}\tau})\upsilon^2, \qquad \omega^\pm := \frac{1}{2}(H \pm \sqrt{H^2 - 4\lambda_1}), \tag{27}$$

where ζ_1 and ζ_2 are the complex numbers, c_1 and c_2 are real, and we assume for definiteness that $\lambda_2 > 0$ and $\lambda_1 < 0$. This is indeed the case for the gravitational field (see below). The constants of integration are uniquely determined by the boundary conditions $x(0) = y$ and $x(s) = x$. Then the Hamilton-Jacobi action takes the form

$$
\begin{aligned}
S = \frac{1}{2}(x_i(s)\dot{x}^i(s) - x_i(0)\dot{x}^i(0)) = \frac{1}{4}(y^i s_{ij} y^j + x^i s_{ij} x^j)\sqrt{H^2 - 4\lambda_1}\operatorname{ctg}\frac{s}{2}\sqrt{H^2 - 4\lambda_1} - \\
- \frac{\sqrt{H^2 - 4\lambda_1}}{2\sin\frac{s}{2}\sqrt{H^2 - 4\lambda_1}}(x^i s_{ij} y^j \cos\frac{s}{2}H + i x^i a_{ij} y^j \sin\frac{s}{2}H) + \\
+ \frac{\sqrt{\lambda_2}}{2}\operatorname{cth} s\sqrt{\lambda_2}[(\upsilon_i^2 y^i)^2 + (\upsilon_i^2 x^i)^2] - \frac{\sqrt{\lambda_2}}{\operatorname{sh} s\sqrt{\lambda_2}} x^i \upsilon_i^2 \upsilon_j^2 y^j = \frac{1}{4} y\sqrt{-f^2 - 4E}\operatorname{ctg}\frac{s}{2}\sqrt{-f^2 - 4E} y + \\
+ \frac{1}{4} x\sqrt{-f^2 - 4E}\operatorname{ctg}\frac{s}{2}\sqrt{-f^2 - 4E} x - y\frac{\sqrt{-f^2 - 4E} e^{\frac{s}{2}f}}{2\sin\frac{s}{2}\sqrt{-f^2 - 4E}} x. \quad (28)
\end{aligned}
$$

Shifting the variables in the initial action (25), it is easy to generalize the above result to the case where the Lagrangian contains the additional terms E_0 and $b_i x^i$. And so, for the system with the Hamilton function

$$
H(p, x) = (p_i + A_i)^2 - E_0 - b_i x^i - \frac{1}{2} E_{ij} x^i x^j, \quad (29)
$$

we have the Hamilton-Jacobi action

$$
S = \frac{1}{4}(x - y)\varkappa \operatorname{ctg} s\varkappa (x - y) + \tilde{y}\frac{\varkappa}{2}(\operatorname{ctg} s\varkappa - \frac{e^{sf}}{\sin s\varkappa})\tilde{x} - \frac{s}{2} b E^{-1} b + \frac{1}{2} b E^{-1} f(x - y) + sE_0, \quad (30)
$$

where $\varkappa := \sqrt{-f^2 - 2E}$, while $\tilde{x}_i = x_i + E_{ij}^{-1} b_j$ and $\tilde{y}_i = y_i + E_{ij}^{-1} b_j$. Note that the factor 1/2 is absent at the kinetic term in the Hamilton function (29). Therefore we have to stretch the proper-time and the potential in (25) accordingly in order to get (29) and (30). Also notice that the Hamilton-Jacobi action (30) contains the inverse matrix E_{ij}^{-1}, but it has a finite limit when E_{ij} becomes degenerate as seen from (30) expanded in s

$$
\begin{aligned}
S = \frac{(x - y)^2}{4s} + \frac{1}{2} x f y + s\Big[\frac{1}{12}(x - y) f^2 (x - y) + \frac{1}{6}(xEx + yEy + xEy) + \frac{1}{2} b(x + y) + E_0\Big] - \\
- \frac{s^2}{6}\tilde{x} E f \tilde{y} - \frac{s^3}{12}\Big[\frac{1}{15}(x - y)(f^2 + 2E)^2 (x - y) + \tilde{x} E^2 \tilde{y}\Big] + \ldots \quad (31)
\end{aligned}
$$

All the terms with E_{ij}^{-1} cancel out. The van Vleck determinant is written as

$$
\begin{aligned}
\det\frac{\partial^2 S}{\partial x^i \partial y^j} = (-2s)^{-d}\det\frac{s\varkappa e^{sf}}{\sin s\varkappa} = (-2s)^{-d}\det\Big(\frac{\sin s\varkappa}{s\varkappa}\Big)^{-1}, \\
-\frac{1}{2}\ln\frac{\sin s\varkappa}{s\varkappa} = -\frac{s^2}{12}(f^2 + 2E) + \frac{s^4}{360}(f^2 + 2E)^2 + \ldots,
\end{aligned} \quad (32)
$$

where we have used the unimodular property of the matrix e^{sf}.

Now we return to our problem. The leading (Gaussian) contribution to the low energy expansion of the heat kernel is obtained [13] if we expand the Hamiltonian (6) in the momentum and coordinate operators up to the second order and throw away the higher terms. Then we solve the Heisenberg equations for the retained quadratic Hamiltonian and find the evolution operator (the heat kernel) in the Fock proper-time [13, 22, 31]. As well-known, this problem is essentially classical for the quadratic Hamiltonians, and the quasiclassical answer,

$$\tilde{G}(\omega, s; \mathbf{x}, \mathbf{y}) = \langle \mathbf{x} | e^{-is[-\tilde{H}(\omega)]} | \mathbf{y} \rangle = \Big[(-2\pi i)^{-d} \det \frac{\partial^2 S}{\partial x^i \partial y^j} \Big]^{1/2} e^{iS(s;x,y)}, \tag{33}$$

for the Green function is the exact one. Here tilde reminds us that the Green function is the bi-density rather than the bi-scalar as in Eq. (7). Formula (33) can also be obtained from the path-integral representation of the heat kernel after a Gaussian integration over the fields $x^i(\tau)$ (see, e.g., [23, 27, 32]).

The Hamiltonian for our system is

$$\begin{aligned} -\tilde{H} &= \bar{g}^{-1/4}(p_i + g_i)\sqrt{\bar{g}}\bar{g}^{ij}(p_j + g_j)\bar{g}^{-1/4} + \frac{1}{2}\bar{\nabla}^i h_i + \frac{1}{4}h_i h^i + \frac{\omega^2}{\xi^2} + m^2 = \\ &= (p_i + g_i)\bar{g}^{ij}(p_j + g_j) + \frac{1}{2}\partial_i(\bar{g}^{ij}\partial_j \ln\sqrt{\bar{g}}) + \frac{1}{4}\partial_i \ln\sqrt{\bar{g}}\bar{g}^{ij}\partial_j \ln\sqrt{\bar{g}} + \frac{1}{2}\bar{\nabla}^i h_i + \frac{1}{4}h_i h^i + \frac{\omega^2}{\xi^2} + m^2. \end{aligned} \tag{34}$$

Of course, we want to derive such an approximate expression for the heat kernel associated with this Hamiltonian that preserve all the symmetries of the exact evolution operator: it should be the kernel of a unitary operator, it should be the bi-density function which is invariant with respect to the gradient transformations of the field g_i. The last property guaranties that the corresponding positive-frequency function will be generally covariant under the spacetime transformations. Besides, we want that already the leading (Gaussian) approximation give us the most exact approximation that we can achieve for the heat kernel in the case of the slowly varying fields $\bar{g}^{ij}$ and g_i. So as we need not to evaluate the higher order corrections to it using the perturbation theory, or reduce this work to a minimum.

To this aim, we adopt the following strategy. For any given points $\mathbf{x}$ and $\mathbf{y}$ connected by the geodesic, we pass to the Riemann normal coordinates with the origin at the midpoint $p(\mathbf{x}, \mathbf{y})$. In this frame, we expand the functions entering the Hamiltonian (34) in a Taylor series. This automatically gives us the covariant expressions at any finite order of the expansion. In particular, using the formulas presented in [27, 29] and in Appendix B we find

$$\begin{aligned} &\frac{1}{2}\partial_i(\bar{g}^{ij}\partial_j \ln\sqrt{\bar{g}}) + \frac{1}{4}\partial_i \ln\sqrt{\bar{g}}\bar{g}^{ij}\partial_j \ln\sqrt{\bar{g}} = -\frac{1}{6}\bar{R} - \frac{1}{6}\bar{\nabla}_i \bar{R} x^i + \frac{1}{2}\bar{r}_{ij}x^i x^j + \dots, \\ &\bar{r}_{ij} := \frac{1}{5}\big(\frac{1}{3}\bar{R}_{ik}\bar{R}^k{}_j - \frac{1}{6}\bar{R}^{kl}\bar{R}_{kilj} - \frac{1}{6}\bar{R}_i{}^{mnk}\bar{R}_{jmnk} - \frac{1}{4}\bar{\nabla}^2 \bar{R}_{ij} - \frac{3}{4}\bar{\nabla}_{ij}\bar{R}\big). \end{aligned} \tag{35}$$

In passing from the bi-density to the bi-scalar, one should multiply the former by

$$\bar{g}^{-1/4}(\mathbf{x})\bar{g}^{-1/4}(\mathbf{y}) = \bar{\Delta}^{1/2}(\mathbf{x},\mathbf{y}), \tag{36}$$

where we have used the relation [27]

$$\bar{\Delta}(\mathbf{x},0) = \bar{\Delta}(0,\mathbf{x}) = \bar{g}^{-1/2}(\mathbf{x}), \tag{37}$$

and the composition property of the covariant van Vleck determinant. Also we need the expansion

$$\omega^2\xi^{-2}(x) = \omega^2\xi^{-2}\big(1 - 2h_i x^i + (2h_i h_j - \bar{\nabla}_i h_j)x^i x^j + \ldots\big) =: \omega^2(\xi^{-2} - b_i^{(0)}x^i - \frac{1}{2}E_{ij}^{(0)}x^i x^j). \tag{38}$$

Hereinafter, all the fields and their derivatives are assumed to be taken at the point p, unless otherwise stated. As long as we use the midpoint prescription, $\bar{\sigma}^i = 2x^i = -2y^i$.

Now, in the Hamiltonian (34), we retain the terms which are at most quadratic in x and p (keeping the order that they are written in (34)) and obtain the Gaussian approximation for the heat kernel

$$\begin{aligned}
G(\omega,s;\mathbf{x},\mathbf{y}) = {} & \frac{\bar{\Delta}^{1/2}(\mathbf{x},\mathbf{y})}{(4\pi i s)^{d/2}}\det\Big(\frac{\sin s\varkappa}{s\varkappa}\Big)^{-1/2} e^{iS_0(s,\bar{\sigma}_i) - isM^2},\\
M^2 := {} & m^2 - \frac{1}{6}\bar{R} + \frac{1}{2}\bar{\nabla}^i h_i + \frac{1}{4}h^2 = m^2 - \frac{1}{6}(R + \xi^2 R_{\mu\nu}g^\mu g^\nu) - \frac{1}{4}h^2,\\
S_0 := {} & \frac{1}{4}\bar{\sigma}\varkappa\,\mathrm{ctg}(s\varkappa)\bar{\sigma} - \frac{1}{2}(\frac{\bar{\sigma}}{2} - bE^{-1})\varkappa\big(\mathrm{ctg}\, s\varkappa - \frac{e^{s\omega f}}{\sin s\varkappa}\big)(\frac{\bar{\sigma}}{2} + E^{-1}b) - \frac{s}{2}bE^{-1}b +\\
& + \frac{\omega}{2}bE^{-1}f\bar{\sigma} - s\frac{\omega^2}{\xi^2},
\end{aligned} \tag{39}$$

where for brevity we omit the matrix indices, $\bar{\sigma} \equiv \bar{\sigma}_i$ in what follows, $\varkappa$ is defined as above with the replacement $f \to \omega f$, and

$$\begin{aligned}
E_{ij} &= \omega^2 E_{ij}^{(0)} + E_{ij}^{(2)}, \qquad & b_i &= \omega^2 b_i^{(0)} + b_i^{(2)},\\
E_{ij}^{(2)} &= \bar{\nabla}_{ij}\Big(\frac{1}{4}h^2 - \frac{1}{2}\nabla^\lambda h_\lambda\Big) - \bar{r}_{ij}, \qquad & b_i^{(2)} &= \bar{\nabla}_i\Big[\frac{1}{6}(R + \xi^2 R_{\mu\nu}g^\mu g^\nu) + \frac{1}{4}h^2\Big].
\end{aligned} \tag{40}$$

Expanding the expression for G in the number of derivatives as it was done in the previous section, one can convince oneself, using formulas (31), (32), that all the terms of the expansion (15) are reproduced save the term

$$\frac{\omega s}{6}\bar{\nabla}^j f_{ji}\bar{\sigma}^i = \frac{\omega s}{6}(\nabla^\mu f_{\mu\nu}\bar{\sigma}^\nu + \bar{\sigma}^\nu f_{\nu\mu}h^\mu). \tag{41}$$

It seems this term cannot be obtained from the Gaussian approximation. Such a term may come from the divergence of the gauge field g_i that appears when one uses the qp-ordering of the Hamiltonian (34). However, the qp-ordered Hamiltonian truncated to its quadratic part is not Hermitian

and the evolution operator is not unitary – the property which we desire to preserve. Thus, substituting the expression obtained to (7), we arrive at the central result of the present paper

$$D^{(+)}(x,y) \approx -i\int ds \int_0^\infty \frac{d\omega}{2\pi} e^{-i\omega t-} \frac{\bar{\Delta}^{1/2}(\mathbf{x},\mathbf{y})(1+\omega s\bar{\nabla}^j f_{ji}\bar{\sigma}^i/6)}{|\xi^2|^{1/4}(\mathbf{x})(4\pi is)^{d/2}|\xi^2|^{1/4}(\mathbf{y})} \det\Big(\frac{\sin s\varkappa}{s\varkappa}\Big)^{-1/2} e^{iS_0(s,\bar{\sigma}_i)-isM^2}, \tag{42}$$

where $t \equiv T$ or the Fock gauge (22) based at the midpoint is implied. The higher derivative term (41) could be "exponentiated", but we shall not investigate this possibility.

Some remarks are in order. Due to the noncommutativity of x and p in the kinetic part of the Hamiltonian (34), there is an ambiguity how to single out the quadratic part from it. For example, one could use the Weyl ordering, where the first term in (34) is rewritten (without the gauge fields)

$$p_i\bar{g}^{ij}p_j = \frac{1}{4}(p_ip_j\bar{g}^{ij} + 2p_i\bar{g}^{ij}p_j + \bar{g}^{ij}p_ip_j) + \frac{1}{4}\partial_{ij}\bar{g}^{ij}. \tag{43}$$

Then one extracts the quadratic part from the Hamiltonian and obtains the same associated classical equations of motion as above, but with the additional correction to the potential. In particular, instead of $-\bar{R}/6$ in (35) one will have

$$-\frac{1}{6}\bar{R} + \frac{1}{4}\partial_{ij}\bar{g}^{ij}(0) = -\frac{1}{4}\bar{R}. \tag{44}$$

It is this correction to the potential term which is argued by DeWitt as the correct one (see [23]), when one uses the path-integral representation of the heat kernel with the midpoint prescription. However, with this potential term, one needs to perform the two-loop calculations in order to reproduce the well-known asymptotic of the heat kernel on the diagonal even at the first power of s. The potential with such a property is, of course, quite unsuitable for our purpose outlined above. In fact, changing the ordering prescriptions, one can obtain any number at the scalar curvature in the potential. The conditions, that we impose on the approximate heat kernel, fix the prescription unambiguously. It was proven in [27] that the correction of the form (35) to the potential term sums all the terms of the derivative expansion of the heat kernel, which contain $\bar{R}$ at the given point.

In order to provide a tighter connection of the positive-frequency function (42) with reality, let us write the quantities entering it in the weak field limit for the vacuum solutions of the Einstein equations (for details, see [24] and Appendix A)

$$\begin{gathered} b_i^{(0)} \approx -\frac{r_g}{r^2}n_i, \qquad b_i^{(2)} \approx -\frac{r_g}{4r^5}n_i, \qquad E_{ij}^{(0)} \approx -\frac{r_g}{r^3}(\delta_{ij} - 3n_in_j), \\ \bar{r}_{ij} \approx -\frac{r_g^2}{80r^6}(5\delta_{ij} - 3n_in_j), \qquad E_{ij}^{(2)} \approx -\frac{3r_g^2}{16r^6}(\delta_{ij} - \frac{39}{5}n_in_j), \\ f_{ij} \approx \frac{2r_g}{mr^3}(M_{ij} + \frac{3}{2}n_{[i}M_{j]k}n_k), \qquad [E,f] \approx \omega^2\frac{3r_g^2}{mr^6}n_{(i}M_{j)k}n_k, \end{gathered} \tag{45}$$

where m is a total mass of the gravitating object, M_{ij} is its angular momentum, r_g is the Schwarzschild radius, r is a distance from the gravitating object, and $n_i = x_i/r$. In the second line, the Schwarzschild metric was used for the calculations. We see that the tensor E_{ij} has one positive and two negative eigenvalues as in (26). The relation $[E, f] = 0$, which we also assumed, is fulfilled when the vector of the angular momentum is parallel to the vector of the gravielectric force (see Appendix A for the definition). The above calculations can be easily generalized to the case when the tensor f_{ij} has a canonical form (26) and the tensor E_{ij} is diagonal in the same basis. One should set $\lambda_1 \in \mathbb{C}$ in the solution (27), and the matrix e^{sf} in the Hamilton-Jacobi action (30) is to be placed near $\tilde{x}$. Nevertheless, further we restrict ourself to the case $[E, f] = 0$.

IV. PHYSICAL IMPLICATIONS

In this section we shall discuss some physical implications of the result obtained above. The crucial point is, of course, the fact that the positive-frequency function (42) depends nontrivially on the Killing vector. This dependence stems from the dependence of the Hamiltonian of the scalar field on the Killing vector of the gravitational background. The positive-frequency function is an observable and so it is just a matter of the physical erudition to extract the dependence on the Killing vector from it. We shall find the various expressions for the propagator of the scalar field which are valid in the different regimes: at sufficiently large and extremely small point separations. Besides, we shall derive the exponentially suppressed contributions to the one-loop effective action which depend nontrivially on the vector field ξ^μ. This action is induced by the polarization of the vacuum of scalar particles and represents the analog of the Heisenberg-Euler action [12, 13] in quantum electrodynamics. In particular, we shall obtain the quasiclassical formulas for the Hawking particle production [3] analogous to Schwinger's formulas for the particle production in the constant electromagnetic field. The Unruh effect and the radiation reaction problem on a curved background will be also addressed and the formula for the acceleration determining the Unruh effect will be derived.

The structures appearing in the heat kernel (39) fall naturally into the pieces denoted by the indices 0 and 2. Let us estimate their ratio. Introducing the characteristic scales in the weak field limit

$$\partial_i \sim r^{-1} \sim L^{-1}, \qquad h_i \sim \varepsilon/L, \qquad \bar{R}_{ij} \sim \varepsilon/L^2, \qquad \text{etc.} \tag{46}$$

where $\varepsilon = (1 - \xi^2) \sim r_g/L \ll 1$, we see that the inequalities

$$\omega^2 |b_i^{(0)}| \gg |b_i^{(2)}|, \qquad \omega^2 |E_{ij}^{(0)}| \gg |E_{ij}^{(2)}|, \tag{47}$$

are equivalent to

$$\omega^2 L^2 \gg 1. \tag{48}$$

Near the horizon, $\varepsilon \approx 1$, the estimations (46) changes,

$$\partial_i \sim h_i \sim L^{-1}\xi^{-2}, \qquad \bar{\nabla}_i h_j \sim \bar{R}_{ij} \sim L^{-2}\xi^{-4}, \qquad \text{etc.} \tag{49}$$

and the conditions (47) become

$$\xi^4 \omega^2 L^2 \gg 1, \tag{50}$$

which is valid in the weak field limit as well. Of course, certain combinations of the fields g_i, f_{ij}, $\bar{R}^i{}_{jkl}$, and their derivatives may be unexpectedly small or vanish on the particular solutions to the Einstein equations, but their orders of magnitude are not larger than given above. Loosely speaking, the inequality (50) says that we consider a propagation of the wave packet consisting of the modes with the wavelengths much smaller than the distance to the gravitating object. Therefore, in the range of applicability of the approximation of slowly varying fields, which we assume from the outset, the inequality (50) has to be hold and the estimations (47) are fulfilled.

Now we evaluate the different types of contributions to the integral (42). Stretching the proper-time $s \to s/\omega$ and redefining

$$E_{ij} \to \omega^2 E_{ij}, \qquad b_i \to \omega^2 b_i, \tag{51}$$

we can write the integral in the form

$$\begin{split} D^{(+)}(x,y) = -i \int \frac{ds}{2\pi} \frac{\bar{\Delta}^{1/2}(\mathbf{x},\mathbf{y})(1 + s\bar{\nabla}^j f_{ji}\bar{\sigma}^i/6)}{|\xi^2|^{1/4}(\mathbf{x})(4\pi i s)^{d/2}|\xi^2|^{1/4}(\mathbf{y})} \times \\ \times \int_0^\infty d\omega \omega^{d/2-1} \det\Big(\frac{\sin s\varkappa}{s\varkappa}\Big)^{-1/2} e^{i\omega(S_0(s,\bar{\sigma}_i) - t_-) - isM^2/\omega}, \end{split} \tag{52}$$

where S_0 is defined as in Eq. (39) with $\omega = 1$. In the representation (26),

$$\begin{split} S_0 = \frac{\bar{\sigma}_\perp^2}{8}\sqrt{H^2 - 2\lambda_1}\Big(\operatorname{ctg} s\sqrt{H^2 - 2\lambda_1} + \frac{\cos sH}{\sin s\sqrt{H^2 - 2\lambda_1}}\Big) + \frac{\bar{\sigma}_\parallel^2}{8}\sqrt{2\lambda_2}\operatorname{th}\frac{s}{2}\sqrt{2\lambda_2} + \\ + \frac{b_\perp^2}{2\lambda_1^2}\sqrt{H^2 - 2\lambda_1}\Big(\operatorname{ctg} s\sqrt{H^2 - 2\lambda_1} - \frac{\cos sH}{\sin s\sqrt{H^2 - 2\lambda_1}}\Big) + \frac{b_\parallel^2}{2\lambda_2^2}\sqrt{2\lambda_2}\operatorname{cth}\frac{s}{2}\sqrt{2\lambda_2} - \\ - \frac{1}{2\lambda_1}(\bar{\sigma}_x b_y - \bar{\sigma}_y b_x)\Big(\sqrt{H^2 - 2\lambda_1}\frac{\sin sH}{\sin s\sqrt{H^2 - 2\lambda_1}} - H\Big) - \frac{s}{2}\Big(\frac{b_\perp^2}{\lambda_1} + \frac{b_\parallel^2}{\lambda_2}\Big) - \frac{s}{\xi^2}, \end{split} \tag{53}$$

where υ_2 is assumed to be directed along the z-axis and the projections to this axis are denoted as parallel.

A. The ω-representation

At first, we take the integral over s. The integrand has the three types of singular points in the s-plane: (a) the essentially singular points on the imaginary axis $s = i\pi n/\sqrt{2\lambda_2}$, (b) the essentially singular points on the real axis $s = \pi n/\sqrt{H^2 - 2\lambda_1}$, and (c) the branching point at the origin. The points of the type (a) are responsible for the Hawking particle production, the points of the type (b) describes the vacuum polarization effects, and the point (c) gives the major contribution to the propagator. None of the contributions from these points can be evaluated exactly. However we can make it approximately under the assumption that the estimation (50) holds. We shall expand S_0 near the singular points retaining only the leading terms of the Laurent series. The higher terms of this series give a negligible contribution. This is easy to see if one makes the variable s dimensionless stretching it once more as $s \to s/\omega$.

Introducing the notation

$$l_{12} := \frac{\sqrt{H^2 - 2\lambda_1}}{\sqrt{2\lambda_2}}, \qquad l_{21} := \frac{\sqrt{2\lambda_2}}{\sqrt{H^2 - 2\lambda_1}}, \qquad l_{H1} := \frac{H}{\sqrt{H^2 - 2\lambda_1}}, \qquad l_{H2} := \frac{H}{\sqrt{2\lambda_2}}, \tag{54}$$

the expansions near the points (a) can be cast into the form

$$S_0 \approx \frac{\bar{\sigma}_\parallel^2}{4x} - i\Big[\frac{\bar{\sigma}_\perp^2}{8}\sqrt{H^2 - 2\lambda_1}\Big(\operatorname{cth}\pi n l_{12} + \frac{\operatorname{ch}\pi n l_{H2}}{\operatorname{sh}\pi n l_{12}}\Big) + \frac{b_\perp^2}{2\lambda_1^2}\sqrt{H^2 - 2\lambda_1}\Big(\operatorname{cth}\pi n l_{12} - \frac{\operatorname{ch}\pi n l_{H2}}{\operatorname{sh}\pi n l_{12}}\Big) - \\ - \frac{i}{2\lambda_1}(\bar{\sigma}_x b_y - \bar{\sigma}_y b_x)\Big(\sqrt{H^2 - 2\lambda_1}\frac{\operatorname{sh}\pi n l_{H2}}{\operatorname{sh}\pi n l_{12}} - H\Big) + \frac{\pi n}{\sqrt{2\lambda_2}}(\xi^{-2} + \frac{1}{2}bE^{-1}b)\Big], \tag{55}$$

where x tends to zero and n is an odd number, and

$$S_0 \approx \frac{b_\parallel^2}{\lambda_2^2 x} - i\Big[\frac{\bar{\sigma}_\perp^2}{8}\sqrt{H^2 - 2\lambda_1}\Big(\operatorname{cth}\pi n l_{12} + \frac{\operatorname{ch}\pi n l_{H2}}{\operatorname{sh}\pi n l_{12}}\Big) + \frac{b_\perp^2}{2\lambda_1^2}\sqrt{H^2 - 2\lambda_1}\Big(\operatorname{cth}\pi n l_{12} - \frac{\operatorname{ch}\pi n l_{H2}}{\operatorname{sh}\pi n l_{12}}\Big) - \\ - \frac{i}{2\lambda_1}(\bar{\sigma}_x b_y - \bar{\sigma}_y b_x)\Big(\sqrt{H^2 - 2\lambda_1}\frac{\operatorname{sh}\pi n l_{H2}}{\operatorname{sh}\pi n l_{12}} - H\Big) + \frac{\pi n}{\sqrt{2\lambda_2}}(\xi^{-2} + \frac{1}{2}bE^{-1}b)\Big], \tag{56}$$

when n is an even number. The mass term and the preexponential factor become in these cases

$$sM^2 \approx \frac{i\pi n M^2}{\sqrt{2\lambda_2}}, \qquad s^{-3/2}\det\Big(\frac{\sin s\varkappa}{s\varkappa}\Big)^{-1/2} \approx \frac{e^{-i\pi(n+1)/2}\sqrt{H^2 - 2\lambda_1}(2\lambda_2)^{1/4}}{\operatorname{sh}\pi n l_{12}\operatorname{sh}^{1/2}(x\sqrt{2\lambda_2})}. \tag{57}$$

Recall that the integration contour in the s-plane goes along the real axis below the singularities lying on it. The half-plane (upper or lower), where we should close the contour, can be determined analyzing the asymptotic behavior of the integrand at the large n. In the case when

$$\sqrt{H^2 - 2\lambda_1} > \omega(\xi^{-2} + \frac{1}{2}bE^{-1}b) + M^2/\omega, \tag{58}$$

we can close the contour in the upper half-plane. Otherwise, we have to close the contour in the lower half-plane and the singular points of the types (b) and (c) do not contribute to the positive-frequency function at such ω's. One may say that such modes do not propagate.

For the energies ω satisfying the estimation (50) and much larger than the Compton wavelength, the inequality (58) is fulfilled. Then the contributions of the singularities (a) are suppressed by the Boltzmann-like factor

$$\exp[\omega\frac{\pi n}{\sqrt{2\lambda_2}}(\xi^{-2}+\frac{1}{2}bE^{-1}b)]\approx\exp\Big[\omega\frac{\pi n}{\sqrt{2\lambda_2^{(0)}}}(\xi^{-2}+\frac{1}{2}b^{(0)}E^{-1}_{(0)}b^{(0)})\Big], \tag{59}$$

where $\lambda_2^{(0)}$ is the eigenvalue of the matrix $E^{(0)}$. The coefficient at the energy ω at $n=1$ can be interpreted as the reciprocal temperature of the Hawking radiation. Inasmuch as the one-loop contribution of one bosonic mode to the effective action reads as

$$\Gamma^{(1)}_{1b}=-i\int dx\frac{\sqrt{|g|}}{|\xi^2|}\int_0^\infty d\omega\omega^2 D^{(+)}(\omega,\mathbf{x},\mathbf{x}), \tag{60}$$

these singularities contribute to the imaginary part of the effective action and suppressed by the same exponential factor. The spatial measure in the integral (60) is that measure with respect to which the mode functions are orthogonal. To evaluate the integral over s, we use the formula

$$\int_H\frac{dz}{\mathrm{sh}^{1/2}(z\sqrt{2\lambda_2})}=-\frac{4\sqrt{2\pi}\Gamma(5/4)}{\sqrt{2\lambda_2}\Gamma(3/4)}, \tag{61}$$

where H is the Hankel contour that runs from $+\infty$ a little bit higher than the real axis, encircles the origin, and then goes to $+\infty$ a little bit lower than the real axis. With this integral at hand, we can write the contributions from the singularities of the type (a) for odd n ($n=1$ is the leading contribution) in the form

$$D^{(+)}(\omega,\mathbf{x},\mathbf{x})|_{(a)}\approx -ie^{-i\pi(n/2+1/4)}\frac{(2\lambda_2)^{1/4}\Gamma(5/4)}{\sqrt{2}\pi|\xi^2|^{1/2}\Gamma(3/4)}\frac{l_{12}}{\mathrm{sh}\,\pi n l_{12}}\times$$
$$\times\,\omega^{1/2}e^{\omega\frac{b_\perp^2}{2\lambda_1^2}\sqrt{H^2-2\lambda_1}\left(\mathrm{cth}\,\pi n l_{12}-\frac{\mathrm{ch}\,\pi n l_{H2}}{\mathrm{sh}\,\pi n l_{12}}\right)+\frac{\pi n}{\sqrt{2\lambda_2}}\left(\frac{\omega}{\xi^2}+\frac{\omega}{2}bE^{-1}b-\frac{M^2}{\omega}\right)}. \tag{62}$$

The imaginary part of the correction to the effective action coming from these singularities is positive as it should be. There exists also the exponentially suppressed contribution to the real part of the effective action from these terms. As far as the contributions with even n are concerned, the nonvanishing term at $1/x$ in the expansion (56) leads to a complication in evaluating this integral and the resulting expression is rather huge. So, we do not write it here.

Near the points (b), we have

$$S_0 \approx \frac{1}{x}\Big[\frac{\bar{\sigma}_\perp^2}{8}(1+(-1)^n\cos\pi n l_{H1}) + \frac{b_\perp^2}{2\lambda_1^2}(1-(-1)^n\cos\pi n l_{H1}) - \frac{(-1)^n}{2\lambda_1}(\bar{\sigma}_x b_y - \bar{\sigma}_y b_x)\sin\pi n l_{H1}\Big] -$$
$$- H(-1)^n \sin\pi n l_{H1}(\frac{\bar{\sigma}_\perp^2}{8} - \frac{b_\perp^2}{2\lambda_1^2}) + \sqrt{2\lambda_2}\,\mathrm{cth}\,\pi n l_{21}(\frac{\bar{\sigma}_\parallel^2}{8} + \frac{b_\parallel^2}{2\lambda_2^2}) +$$
$$+ \frac{H}{2\lambda_1}(\bar{\sigma}_x b_y - \bar{\sigma}_y b_x)(1-(-1)^n\cos\pi n l_{H1}) - \frac{\pi n}{\sqrt{H^2-2\lambda_1}}(\xi^{-2} + \frac{1}{2} bE^{-1}b), \tag{63}$$

and

$$sM^2 \approx \frac{\pi n M^2}{\sqrt{H^2-2\lambda_1}}, \qquad s^{-3/2}\det\Big(\frac{\sin s\varkappa}{s\varkappa}\Big)^{-1/2} \approx \frac{(-1)^n(2\lambda_2)^{1/4}}{x\,\mathrm{sh}^{1/2}(\pi n l_{21})}. \tag{64}$$

The integrand of (52) is a single-valued function in the neighbourhood of these singular points because of the degeneracy of the eigenvalue $\sqrt{H^2-2\lambda_1}$. As a result, the integral over s going around these points can be simply evaluated. Assuming the inequality (58) is satisfied, we arrive at

$$D^{(+)}(\omega,\mathbf{x},\mathbf{y})|_{(b)} \approx \frac{\bar{\Delta}^{1/2}(\mathbf{x},\mathbf{y})\Big(1+\frac{\pi n\bar{\nabla}^j f_{ji}\bar{\sigma}^i}{6\sqrt{H^2-2\lambda_1}}\Big)(-1)^n(2\lambda_2)^{1/4}}{|\xi^2|^{1/4}(\mathbf{x})(4\pi i)^{3/2}|\xi^2|^{1/4}(\mathbf{y})\,\mathrm{sh}^{1/2}(\pi n l_{21})}\omega^{1/2} e^{i\omega S_0^f - \frac{i\pi n M^2}{\omega\sqrt{H^2-2\lambda_1}}}, \tag{65}$$

where S_0^f is the finite part of the expansion (63) at $x \to 0$.

The main contribution to the positive-frequency function comes from the origin of the s-plane. From the expansion (31) near this point, we deduce

$$S_0 = \frac{\bar{\sigma}^2}{4s} + s\big[\frac{1}{24}\bar{\sigma}(2f^2+E)\bar{\sigma} - \frac{1}{\xi^2}\big] + \frac{s^2}{6}\bar{\sigma}fb - \frac{s^3}{12}\big[b^2 + \frac{1}{15}\bar{\sigma}(f^2+2E)^2\bar{\sigma} - \frac{1}{4}\bar{\sigma}E^2\bar{\sigma}\big] + \dots \tag{66}$$

Rescaling the proper-time $s \to s/\omega$ and bearing in mind that S_0 is multiplied by ω in the exponent (52), we see that the terms of the above expansion at the second power of s and higher are small provided the condition (50) holds. Besides, there are also the saddle points near (after the rescaling)

$$s^2 = \frac{\omega^2\bar{\sigma}^2}{4(\tilde{E} - \tilde{m}^2/\omega^2)}, \qquad \tilde{E} := -1/\xi^2 + \bar{\sigma}(2f^2 + E^{(0)})\bar{\sigma}/24, \qquad \tilde{m}^2 := M^2 - \bar{\sigma}E^{(2)}\bar{\sigma}/24. \tag{67}$$

If these points are situated near the origin they will considerably contribute to the integral. Here we are interested in the approximation of the propagator for the point separation much larger than the wavelength of a mode (for the extreme case of the infinitely small $\bar{\sigma}$ and t see the next subsection). So, we assume that those extremum points are far from the singular point $s = 0$. This condition is fulfilled when

$$|\xi^2|\omega^2\bar{\sigma}^2 \gg 1. \tag{68}$$

Hence, keeping only the terms at s^{-1}, s^0, and s in the exponent of (52) and the leading contribution from the preexponential factor, we obtain the approximate expression

$$D^{(+)}(x,y)|_{(c)}\approx\frac{-i\bar{\Delta}^{1/2}(\mathbf{x},\mathbf{y})}{|\xi^2|^{1/4}(\mathbf{x})|\xi^2|^{1/4}(\mathbf{y})}\int\frac{ds}{2\pi}\int_0^\infty d\omega\frac{\omega^{d/2-1}}{(4\pi is)^{d/2}}e^{i\omega(\frac{\bar{\sigma}^2}{4s}-t_-+s\tilde{E})-is\tilde{m}^2/\omega}. \tag{69}$$

The integral over s is reduced to the Bessel function $J_{1/2}$ for $d=3$ and, consequently, is expressed in terms of elementary functions. After a little algebra, we find

$$D^{(+)}(x,y)|_{(c)}=\frac{-i\bar{\Delta}^{1/2}(\mathbf{x},\mathbf{y})}{|\xi^2|^{1/4}(\mathbf{x})|\xi^2|^{1/4}(\mathbf{y})}\int_0^\infty\frac{d\omega}{4\pi^2|\bar{\sigma}|}\theta(\omega^2\tilde{E}-\tilde{m}^2)e^{-i\omega t_-}\sin(|\bar{\sigma}|\sqrt{\omega^2\tilde{E}-\tilde{m}^2}). \tag{70}$$

The latter integral is the same as the integral for the positive-frequency function in a flat spacetime at $\tilde{m}^2\geq 0$ with the obvious redefinitions. Applying the formulas from the appendix of [33], we come to

$$\begin{aligned}
D^{(+)}|_{(c)}=&\frac{-\bar{\Delta}^{1/2}(\mathbf{x},\mathbf{y})}{|\xi^2|^{1/4}(\mathbf{x})\tilde{E}^{1/2}|\xi^2|^{1/4}(\mathbf{y})}\Big\{\frac{\operatorname{sgn}(t)}{4\pi}\delta(\lambda)+\\
&+\frac{i\tilde{m}\theta(\lambda)}{8\pi\sqrt{\lambda}}\big[N_1(\tilde{m}\sqrt{\lambda})+i\operatorname{sgn}(t)J_1(\tilde{m}\sqrt{\lambda})\big]+\frac{i\tilde{m}\theta(-\lambda)}{4\pi^2\sqrt{-\lambda}}K_1(\tilde{m}\sqrt{-\lambda})\Big\}\approx\\
\approx&\frac{-\bar{\Delta}^{1/2}(\mathbf{x},\mathbf{y})}{|\xi^2|^{1/4}(\mathbf{x})\tilde{E}^{1/2}|\xi^2|^{1/4}(\mathbf{y})}\Big[\frac{\operatorname{sgn}(t)}{4\pi}\delta(\lambda)-\frac{i}{4\pi^2\lambda}+\frac{i\tilde{m}^2}{8\pi^2}\ln\frac{\tilde{m}|\lambda|^{1/2}}{2}-\frac{\tilde{m}^2}{16\pi}\operatorname{sgn}(t)\theta(\lambda)\Big],\\
&\lambda\equiv-\tilde{X}^2(x,y):=\tilde{E}^{-1}t^2-\bar{\sigma}^2,
\end{aligned} \tag{71}$$

where the last approximate expression is the expansion of the positive-frequency function at the small mass $\tilde{m}$. Of course, the positive-frequency function obtained can be written in a more compact form with the $i\varepsilon$-prescription $t\to t_-$. The unfolded form presented in (71) allows us to see better the structure of its singularities. It is clear that all the other Green functions can be obtained from the positive-frequency function.

Such an expression for the main contribution to the positive-frequency function has several physical implications. First, we see that the wave packet of scalar particles of the mass m in a slowly varying gravitational field behaves like a massive particle with the mass $\tilde{m}$ given in (67). In particular, if we neglect the term in $\tilde{m}^2$ proportional to $\bar{\sigma}^2$ and consider the vacuum solution to the Einstein equations then the mass squared acquires the shift which is equal to $-h^2/4$ (for an analogous but not the same effect in quantum electrodynamics see [14]). This is a tiny negative quantity and is of the order of the Unruh temperature squared. One could expect the appearance of such a correction to the mass already from the expression (6) of the Fourier transformed Klein-Gordon operator. Notice that such a correction to the mass squared is also necessary to reproduce the standard asymptotic expansion of the heat kernel (18) and the propagator (see Eq. (96) below). This correction makes no trouble for the massive particles, but for the massless scalar particles it

seems result in the tachyonic dispersion law in the adapted system of coordinates and to the instability of a vacuum. If $\tilde{m}^2 < 0$ and small, the additional correction appears in the positive-frequency function

$$\delta D^{(+)}(x,y)|_{(c)} \approx \frac{-\bar{\Delta}^{1/2}(\mathbf{x},\mathbf{y})}{|\xi^2|^{1/4}(\mathbf{x})\tilde{E}^{1/2}|\xi^2|^{1/4}(\mathbf{y})}\frac{\tilde{m}^2}{4\pi^2}, \tag{72}$$

which comes from the accurate evaluation of the integral (70) and should be added to the expansion in the third line of (71). Certainly, we should not regard the expression (71) too seriously for the modes with the energies squared of the order of h^2 where the tachyonic effects may appear. These modes are out of the range of the applicability of the approximation we made in deriving (71). However, for the massive particles and for the high energy modes of the massless ones satisfying (50), (68) the positive-frequency function in the representations (70) or (71) with the corrections (62) and (65) describes a propagation quite well.

Second, the effective interval $\tilde{X}^2$ does not coincide with the interval (20). Indeed, it is easy to see for the worldline of a particle staying at rest, $\bar{\sigma} = 0$, that the equation $X^2(\tau) = 0$ has the nontrivial complex solutions (see Eqs. (20), (24)), while the equation $\tilde{X}^2(\tau) = 0$ has not. If we assume

$$|\xi^{-2}| \gg |\bar{\sigma}(2f^2 + E^{(0)})\bar{\sigma}/24|, \tag{73}$$

then

$$\tilde{X}^2 \approx \xi^2 t^2 + \frac{\xi^4 t^2}{24}\bar{\sigma}(2f^2 + E^{(0)})\bar{\sigma} + \bar{\sigma}^2. \tag{74}$$

The latter expression differs from (20) only by the terms in the square brackets which contain T. It readily follows from this observation the expansion of $\tilde{X}^2$ on an arbitrary worldline:

$$\tilde{X}^2 \approx -\tau^2 - \frac{1}{12}a_U^2\tau^4 + \dots, \qquad a_U^2 := (\nabla_u u)^2 - [(a_\mu^{gm} + a_\mu^{ge})^2 - a_{gm}^2], \tag{75}$$

where $a_\mu^{gm} := (\xi u)f_{\mu\nu}u^\mu$ and $a_\mu^{ge} := (\xi u)(gu)h_\mu$ are the gravimagnetic and gravielectric forces divided by the mass of a particle, respectively (see Appendix A for details). In particular, the acceleration squared a_U^2 is zero for a rest particle when $u^\mu = \xi^\mu|\xi^2|^{-1/2}$. We shall call the acceleration a_U as the Unruh acceleration since it determines the Unruh effect for a detector moving in a curved spacetime.

As well known (see, e.g., [4, 23]), the transition rate of the detector to the excited state is determined by the Fourier transform of the positive-frequency function taken on the worldline. More precisely, for the two points $x(\tau_1)$ and $x(\tau_2)$ we introduce $\bar{\tau} := (\tau_1 + \tau_2)/2$ and $\tau := \tau_1 - \tau_2$,

and write $D^{(+)}(x(\tau_1), x(\tau_2)) = D^{(+)}(\bar{\tau}, \tau)$. Then the rate of transition of the detector to the excited state $\omega > 0$ is proportional to

$$\int d\tau e^{-i\omega\tau} D^{(+)}(\bar{\tau}, \tau), \tag{76}$$

where the integration contour in the τ-plane passes slightly below the real axis. The positive-frequency function $D^{(+)}(\tau)$ is singular at $\tau = 0$, where it possesses the pole of second order (for the non-isotropic worldline). All its other singularities are arranged symmetrically with respect to the imaginary axis. Therefore, up to the fourth order in τ, they lie only on the real or imaginary axes when $a_U^2 < 0$ and $a_U^2 > 0$, respectively. In the former case, the Unruh detector feels nothing since we can close the integration contour in the lower half-plane. Of course, there may be singularities that are not catched by the expansion (75), but they lie far from the real axis and so they are suppressed. In the case $a_U^2 > 0$, the Unruh detector will detect the excitations at the reciprocal temperature

$$\beta_U = \frac{2\pi}{a_U}, \tag{77}$$

where we have replaced the factor $\sqrt{12}$ by 2π matching the formula for the Unruh temperature with its flat spacetime analog for the hyperbolic motion. A concrete value of this factor depends on a character of the particle motion and requires a more detailed information about its worldline. Nevertheless, formula (77) provides a good approximation for the temperature by the order of magnitude. Thus we see that the Unruh detector can be employed to test the variations of the field ξ^μ. Notice that if we used the interval (20) in order to describe the Unruh effect, we would obtain $(\nabla_u u)^2$ for the Unruh acceleration and, consequently, arrive at the unphysical result that the resting Unruh detector gets excited in a stationary gravitational field. We should stress the important difference between the Hawking [3] and the Fulling-Unruh [4, 34] effects. The Hawking particle production is caused by the action of the gravitational forces, and the detector at rest may records this process. Contrarily, the Unruh detector responds to the action of the inertial forces. The latter can be defined as a 4-vector (see Appendix A) as long as the vector field ξ^μ exists on the manifold.

The fact that the effective interval (74) differs from the geodetic interval results also in a nonvanishing local expression for the radiation reaction force acting on the charged particle in a free fall. It is not difficult to show (see, e.g., [5–8]) that the retarded Green function constructed from the propagator with the interval (20) (see Eq. (96) below) yields the local contribution to the radiation

reaction force proportional to

$$\nabla_u \nabla_u u_\mu - (\nabla_u u)^2 u_\mu, \tag{78}$$

on the vacuum solutions to the Einstein equations. This contribution vanishes for the geodesic motion and is not zero for a particle at rest. As for the Green function with the effective interval (75), the situation will be reversed: the radiation reaction force acting on the particle at rest will be zero, and it will be nonzero for the particle in a free fall. In a certain sense, the additional terms appearing in the effective equations of motion of a particle and stemming from the difference of (71) from the DeWitt anzats take effectively into account the so-called tail term of the radiation reaction force. We postpone a thorough investigation of this problem to a future research, but should note that the charged particles can be also used to detect variations of the vector field ξ^μ.

One may wonder why the retarded (advanced) Green function following from (71) does not satisfy the general theorems concerning the structure of singularities of the fundamental solution to the hyperbolic partial differential equation [35, 36]. The singularities of such a solution must lie on the characteristic cone determined by the geodetic interval, but we saw that the effective interval (74) does not coincide with this interval. The answer is that the expression for the main contribution to the positive-frequency function (71) is valid only for the point separation much larger than the wavelengths of the modes in the wave packet (see Eq. (68)). And so, the general theorem does not apply to the expression (71). In order that the wave packet propagates along the characteristic surface, it should be delta-shaped in the space at the initial moment and consists of the ultrarelativistic modes. However, the delta-shaped wave packet is infinitely broad in the frequency space and does not obey the restriction (68). From the uncertainty relation for massless particles

$$\Delta\omega \Delta x \geq 2\pi, \tag{79}$$

where $\Delta\omega$ is a width of the wave packet in the frequency space and Δx is its width in the space, we find that the above expression (71) for the Green function holds only for the wave packets with

$$\Delta x \gg \frac{2\pi}{\omega_0}, \tag{80}$$

where ω_0 is the central frequency of the wave packet of the ultrarelativistic (massless) particles. Roughly speaking, the deviance of the singular surface of the contribution (71) to the positive-frequency function from the geodetic light cone just indicates that the wave packet with the extension much larger than the wavelengths of its modes moves slightly different than an ideal point

massless particle. In the next subsection we shall see that in the limit $(t, \bar\sigma) \to 0$ (violating the condition (68)) the structure of singularities of the positive-frequency function (52) is such as dictated by the theorems.

B. The s-representation

Hitherto we have analyzed the ω-representation of the positive-frequency function and derived, in particular, the expression for the major contribution to it using this representation. It turns out that under the assumption (50) we can take the integral over ω in (52) making the additional reasonable approximations. Thereby we shall derive the s-representation of the positive-frequency function. This representation will be employed to obtain the standard asymptotic expansion of the propagator in terms of the geodetic interval squared 2σ at $(t, \bar\sigma) \to 0$.

If the condition (50) is satisfied, we can expand S_0 staying in the exponent in formula (52) in the series in ω and retain only the terms at ω and ω^{-1}. This can be easily done under the assumption that

$$[E^{(2)}, E^{(0)}] \approx 0, \qquad [E^{(2)}, f] \approx 0, \tag{81}$$

The latter relations are valid in the weak field limit as well as for the spherically symmetric metrics. Then S_0 in (52) keeps its form with the replacements $E \to E^{(0)}$ and $b \to b^{(0)}$, while M^2 acquires the correction. The expression for this correction is rather huge. At the coincidence limit it reads as

$$\begin{split}\delta M^2|_{\bar\sigma=0} = b^{(2)} E^{-1}_{(0)} b^{(0)} - \frac{1}{2} b^{(0)} E^{-1}_{(0)} E_{(2)} E^{-1}_{(0)} b^{(0)} + \frac{1}{2} b^{(0)} E^{-1}_{(0)} \frac{E_{(2)}}{\sin^2 s\varkappa} (\cos s\varkappa \operatorname{ch} sf - 1) E^{-1}_{(0)} b^{(0)} - \\ - \frac{1}{s} \Big[b^{(2)} E^{-1}_{(0)} \varkappa + b^{(0)} E^{-1}_{(0)} E_{(2)} \big(E^{-1}_{(0)} f^2 + \frac{3}{2} \big) \varkappa^{-1} \Big] \big(\operatorname{ctg} s\varkappa - \frac{\operatorname{ch} sf}{\sin s\varkappa} \big) E^{-1}_{(0)} b^{(0)},\end{split} \tag{82}$$

where $\varkappa$ is defined as before, but with $E \to E^{(0)}$. At the small s, this correction behaves as

$$\delta M^2 = -\frac{\bar\sigma E^{(2)} \bar\sigma}{24} + O(s\bar\sigma). \tag{83}$$

This expression is valid even if Eqs. (81) do not hold. The preexponential factor in (52) does not change apart from the redefinition of $\varkappa$. Then the integral over ω is reduced to the Hankel function

$$\int_0^\infty d\omega \omega^{d/2-1} e^{i\omega(S_0 - t_-) - is\bar m^2/\omega} = i^{d+1} \pi (is)^{d/2} \Big(\frac{\bar m^2}{a} \Big)^{d/4} H^{(1)}_{d/2} (2\bar m a^{1/2}), \tag{84}$$

where $\bar m^2 := M^2 + \delta M^2$, the square root in the argument of the Hankel function has the cut along

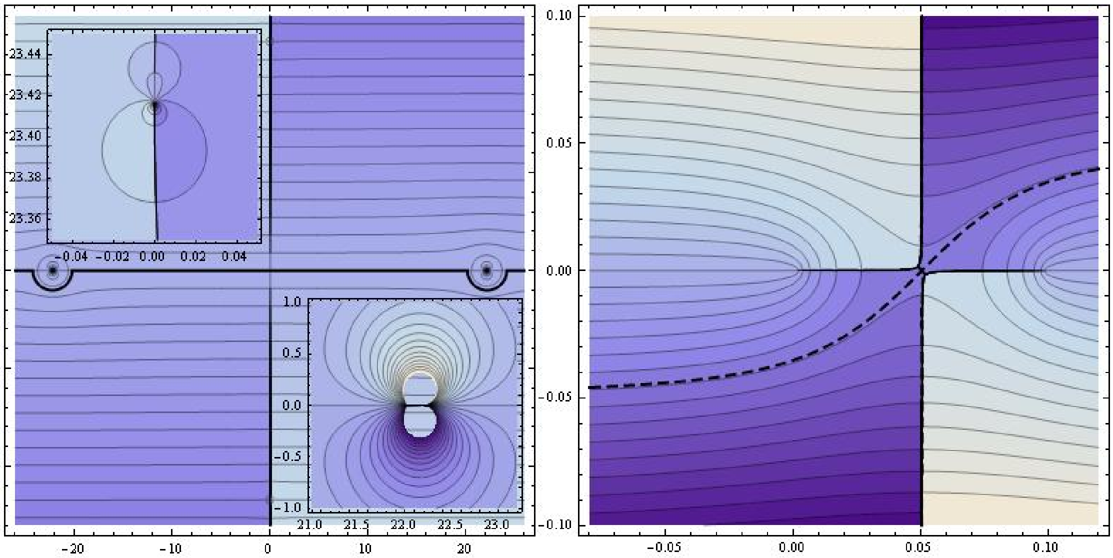

Figure 1. The typical contour plot of the imaginary part of the function staying in the argument of the Hankel function in Eq. (87). The imaginary part changes its sign when reflected in the real axis. The insets depict the structure of singularities of the types (a) and (b) and the cuts near them. The thin lines on all these plots are the lines of the steepest descent. On the left panel: The thick line going along the real axis is the initial integration contour. On the right panel: The structure of singularities near the origin is depicted. The dashed line shows the steepest descent contour. The ends of the cuts lying on the real axis are the zeroes of $a(s)$ nearest to the origin. The small line connecting two cuts intersects the integration contour at the saddle point.

the positive real semi-axis, and $a := s(t_- - S_0)$,

$$a = s^2(\xi^{-2} + \frac{1}{2}bE^{-1}b) - s(t_- + \frac{1}{2}bE^{-1}f\bar{\sigma}) - \\ - \bar{\sigma}\frac{s\varkappa}{4}\operatorname{ctg} s\varkappa\bar{\sigma} + (\frac{\bar{\sigma}}{2} - bE^{-1})\frac{s\varkappa}{2}\big(\operatorname{ctg} s\varkappa - \frac{e^{sf}}{\sin s\varkappa}\big)(\frac{\bar{\sigma}}{2} + E^{-1}b). \quad (85)$$

In our case the Hankel function is expressed in terms of the elementary functions

$$\Big(\frac{\bar{m}^2}{a}\Big)^{d/4} H^{(1)}_{d/2}(2\bar{m}a^{1/2}) = -\frac{i + 2\bar{m}a^{1/2}}{2\pi^{1/2}a^{3/2}} e^{2i\bar{m}a^{1/2}}. \quad (86)$$

Thus the positive-frequency function can be written as

$$D^{(+)}(x,y) = i^d \int \frac{ds\bar{\Delta}^{1/2}(\mathbf{x},\mathbf{y})(1 + s\bar{\nabla}^j f_{ji}\bar{\sigma}^i/6)}{2|\xi^2|^{1/4}(\mathbf{x})(4\pi)^{d/2}|\xi^2|^{1/4}(\mathbf{y})} \det\Big(\frac{\sin s\varkappa}{s\varkappa}\Big)^{-1/2}\Big(\frac{\bar{m}^2}{a}\Big)^{d/4} H^{(1)}_{d/2}(2\bar{m}a^{1/2}). \quad (87)$$

The singularities of the integrand are located at the points (a) and (b) discussed above and also at the points where $a(s) = 0$. The cuts stemming from the square roots in the Hankel function and the determinant stretch between these singular points as depicted on Fig. 1. Recall that the integration contour in the s-plane lies a little bit lower than the real axis.

Unfortunately, the integral (87) cannot be evaluated exactly. Therefore we shall calculate it by the steepest descent method. To this end, we have to deform the integration contour and rise it from the forth quadrant to the first one (see Fig. 1, the right panel). This gives the additional contributions to the integral from the cuts located on the positive real semi-axis, which are responsible for the vacuum polarization effects. We have already evaluated such contributions in the previous subsection and so here we concentrate on the major contribution to the positive-frequency function and its singularities.

The main contribution to the integral (87) comes from the extremum positioned near the point $s = 0$. The singularities of the positive-frequency function appear when the two branching points nearest to the origin (zeroes of $a(s)$) approach the saddle point and pinch the integration contour. At $(t, \bar{\sigma}) \to 0$, this saddle point and the two branching points tend to the origin. In this limit the correction to M^2 is small and $\bar{m}$ can be regarded as a constant. Hence, these saddle and branching points are determined solely by the function $a(s)$. In the vicinity of the extremum we introduce a new variable z such that

$$z^2 + z_0^2 = -a(s), \tag{88}$$

where z_0 is a constant to be determined and $z = 0$ corresponds to the extremum point. Making use of the expansion (66), we have

$$a = -\frac{\bar{\sigma}^2}{4} + st - s^2\tilde{E} - \frac{s^3}{6}b^{(0)}f\bar{\sigma} + \frac{s^4}{12}\big[b_{(0)}^2 + \frac{1}{15}\bar{\sigma}(f^2 + 2E^{(0)})^2\bar{\sigma} - \frac{1}{4}\bar{\sigma}E_{(0)}^2\bar{\sigma}\big] + O(s^5), \tag{89}$$

where we imply that t has a small negative imaginary part. Now the extremum s_0 of the function $a(s)$ can be found perturbatively assuming that $t \sim \bar{\sigma} \sim l \to 0$, i.e., we suppose that l is the smallest scale in the problem. Simple calculations give

$$s_0 = \frac{t}{2\tilde{E}} + \frac{3}{8}\frac{c_3 t^2}{\tilde{E}^3} + \big(\frac{9}{16}\frac{c_3^2}{\tilde{E}^5} + \frac{c_4}{4\tilde{E}^4}\big)t^3 + \dots, \tag{90}$$

where c_3 and c_4 are the coefficient of the expansion of the function $a(s)$ at s^3 and s^4, respectively. At the saddle point the function $a(s)$ is proportional to the geodetic interval squared

$$a(s_0) = -\frac{1}{4}\Big\{\bar{\sigma}^2 + \xi^2 t^2 + \frac{1}{12}\xi^4 t^2\Big[\bar{\sigma}\big(f^2 + \xi^{-2}(\bar{\nabla}_i h_j - h_i h_j)\big)\bar{\sigma} - 2hf\bar{\sigma}t - h^2t^2\Big]\Big\} + \dots = -\frac{1}{4}\sigma_\mu\sigma^\mu. \tag{91}$$

This determines the constant z_0.

Carrying out the calculations, we shall keep the terms in (87) up to the second order in l only. This will allow us to find the singular and finite parts of the positive-frequency function at $l \to 0$.

In particular, according to this approximation $\bar{m}$ should be replaced by M as seen from (83) and (87). Thus we have

$$\begin{gathered}\dot{z}\approx\sqrt{\tilde{E}-3c_3s_0-6c_4s_0^2},\qquad \ddot{z}\approx-\frac{c_3+4c_4s_0}{\dot{z}},\qquad \dddot{z}\approx-\frac{3c_4}{\dot{z}}-\frac{3}{4\dot{z}^3}(c_3+4c_4s_0)^2,\\ s\approx s_0+\frac{z}{\dot{z}}+\frac{c_3+4c_4s_0}{2\dot{z}^4}z^2+\big[c_4+\frac{7}{12}(c_3+4c_4s_0)^2\big]\frac{z^3}{2\dot{z}^5},\end{gathered}\tag{92}$$

where the overdots denote the derivatives with respect to s taken at $s=s_0$. Introducing the notation A_0, A_1, and A_2 for the coefficients of the expansion in s of the preexponential factor in (87), we can write

$$\begin{split}\frac{ds}{dz}(A_0+A_1s+A_2s^2)\approx\frac{A_0+A_1s_0+A_2s_0^2}{\dot{z}}+&\\ +\big[A_1+2A_2s_0+(A_0+A_1s_0)\frac{c_3+4c_4s_0}{\dot{z}^2}\big]\frac{z}{\dot{z}^2}&+(A_2+\frac{3c_4}{2\dot{z}^2}A_0)\frac{z^2}{\dot{z}^3}.\end{split}\tag{93}$$

The terms at the odd powers of z can be omitted as long as the integration over z is carried out on the symmetric interval. As a result, the integral in (87) can be cast into the form

$$\begin{split}2\frac{\partial}{z_0\partial z_0}\int_0^\infty dz\Big[\frac{A_0+A_1s_0+A_2s_0^2}{\dot{z}}+(A_2+\frac{3c_4}{2\dot{z}^2}A_0)\frac{z^2}{\dot{z}^3}\Big]\frac{e^{-2M\sqrt{z^2+z_0^2}}}{\sqrt{z^2+z_0^2}}=&\\ =\frac{4M}{\dot{z}z_0}(A_0+A_1s_0+A_2s_0^2)K_1(2Mz_0)+\frac{2}{\dot{z}^3}(A_2+\frac{3c_4}{2\dot{z}^2}A_0)K_0(2Mz_0).&\end{split}\tag{94}$$

Expanding $\bar{\Delta}^{1/2}$ and ξ^2 in (87) near the midpoint up to the second order in l, we arrive at

$$D^{(+)}(x,y)\approx\frac{-i}{32\pi^2}\Big\{[1+\frac{1}{12}R_{\mu\nu}(t\xi^\mu+\bar{\sigma}^\mu)(t\xi^\nu+\bar{\sigma}^\nu)]\frac{4M}{z_0}K_1(2Mz_0)-[\frac{\xi^2}{6}f^2+h^2-\frac{2}{3}\nabla_\lambda h^\lambda]K_0(2Mz_0)\Big\}.\tag{95}$$

Finally, we expand the Macdonald functions at the small argument [37]

$$D^{(+)}(x,y)\approx\frac{-i}{8\pi^2}\Big\{\frac{1+\frac{1}{12}R_{\mu\nu}\sigma^\mu\sigma^\nu}{\sigma}+\frac{1}{2}(m^2-\frac{1}{6}R)\ln\frac{e^{2\gamma}M^2\sigma}{2}-\frac{M^2}{2}\Big\},\tag{96}$$

where γ is the Euler constant and, as before, σ is the world function, σ_μ is its derivative.

We see that the singularities of the retarded and advance Green functions lie on the geodetic light cone as it should be according to the general theorems. The expansion of these functions in terms of σ coincides with the standard one (see [21, 23]) and is independent of ξ^μ. Whereas the imaginary part of the propagator depends on the Killing vector nontrivially and "remembers" the vacuum state with respect to which the propagator is defined. We also see that the correction to the mass squared is reproduced again. Without it, in particular, the factor at the logarithm in (96) would not have the the covariant form independent of ξ^μ. The mention should be made that the positive-frequency function (96) cannot be employed to analyze the Unruh effect or the radiation

reaction problem. For these problems the main contribution to the corresponding integrals comes from the sufficiently large point separation (smaller than L, but larger than the typical wavelength) where the expansions bringing us to (96) are not valid. The latter expansions may be justified in analyzing these problems only in the case of the extreme accelerations a caused by the external (nongravitational) force. In this case, the relevant contributions come from the point separations of the order of $l \sim a^{-1}$ and the other terms entering the Unruh acceleration (75) can be neglected.

Concluding this section, we write out the s-representation for the one-loop correction to the effective action induced by one bosonic mode. Comparing the formulas (52) and (60), one readily deduces

$$\Gamma^{(1)}_{1b} = i \int dx \frac{\sqrt{|g|}}{|\xi^2|} \partial_t^2 D^{(+)}(t, \mathbf{x}, \mathbf{x})|_{t=-i\Lambda^{-1}}, \tag{97}$$

where Λ is the ultraviolet energy cutoff. From the integral representation of the Hankel function (84) we observe that the differentiation with respect to t just results in the change of the overall sign and the replacement $d \to d+4$ in (87) apart from the factor $4\pi i s$. Consequently, we have

$$\Gamma^{(1)}_{1b} = \int dx \sqrt{|g|} \int \frac{i^{d+1} ds s^2}{2|\xi^2|^{3/2}(4\pi)^{d/2}} \det\Big(\frac{\sin s\varkappa}{s\varkappa}\Big)^{-1/2} \Big(\frac{\bar{m}_0^2}{a}\Big)^{d/4+1} H^{(1)}_{d/2+2}(2\bar{m}_0 a^{1/2}), \tag{98}$$

where $\bar{m}_0$ equals $\bar{m}$ at $\bar{\sigma} = 0$ and $a(s)$ is given by the expression (85) with $\bar{\sigma} = 0$ and $t_- = -i\Lambda^{-1}$.

V. DISCUSSION

In the previous sections we obtained an approximate solution to a purely mathematical problem of finding the Green function for a scalar quantum field on a stationary background under certain assumptions on the external fields (the Boulware vacuum is implied). We have shown that the two-point Green function of a scalar field on a curved background and the induced effective action (in particular, its imaginary part) depend nontrivially on the Killing vector ξ^μ, i.e., this Killing vector field enters explicitly into the expressions and these expressions cannot be rewritten in a local form in terms of the 4-metric and its curvature alone. This dependence stems from the form of the Schrödinger equation for the quantum fields on a curved background

$$i\partial_t \Psi = \int d\mathbf{x} \sqrt{|g|} \Big[\xi^2 g_\mu T^{\mu\nu} g_\nu + \nabla_\mu \big(\phi[\xi^2 g^\mu g^\nu - \frac{1}{2} g^{\mu\nu}]\partial_\nu \phi\big)\Big]\Psi, \tag{99}$$

where Ψ is the state of quantum scalar fields, t is the time coordinate, and $T^{\mu\nu}$ is the (regularized) operator of the energy-momentum tensor in the Schrödinger representation. The second term in the right-hand side of Eq. (99) is needed in order to throw the time derivatives to one of the fields

ϕ entering the energy-momentum tensor. Then the Hamiltonian is diagonalized in the basis of the mode functions associated with (6). The Schrödinger equation explicitly depends on the field g_μ and the stationary states of this equation, in particular the vacuum state, also depend on it. For a flat spacetime with the vacuum state corresponding to the zero eigenvalue this dependence disappears, at least formally, because of the global Poincaré symmetry of the Schrödinger equation.

Assuming that the energy-momentum tensor of the matter fields is covariantly divergenceless in the case of a nonstationary metric [38, 39], we arrive at the dynamical equations of a hydrodynamic form [9] for the vector field ξ^μ with appropriate boundary and initial conditions. The form of these equations strongly suggests that the field g_μ should be quantized. In this case, Eq. (99) is explicitly generally covariant as easy to see integrating it over t. The quantization of the hydrodynamic equations is more or less known [40] and is used in describing the cosmological fluctuations (see, e.g., [41, 42]). The quantum field $\hat{g}_\mu$ is decomposed into the condensate part g_μ and the fluctuations $\delta\hat{g}_\mu$. Then the latter is represented in terms of the creation and annihilation operators, the particles being identified with the phonons. The properties of these phonons follow from the fluid energy-momentum tensor. In particular, as found in [9], the sound speed squared of such phonons equals $-\varphi_N/\alpha$ in the weak field limit, where φ_N is the Newtonian potential and α is the power of decay of the induced energy-momentum tensor at large distances from a gravitating body, i.e., $T_{\mu\nu} \sim r^{-\alpha}$. In terms of the Feynman diagrams, the phonons $\delta\hat{g}_\mu$ appear in the graphs. To say figuratively, it looks like the particles ring in moving in the condensate of the field $\hat{g}_\mu$.

Although we do not touch in this paper the problem of quantization of the gravitational field, it should be noted that one or another solution to the time problem is necessary for the gravity quantization programme (for a review, see [43]). The vector field ξ^μ and its dual, when integrable, can be employed to define the so-called world time [24] and provide the possible preferred definition of time. Such a method to solve this problem is not new and is known as the reference fluid approach (see, e.g., [43–46]). A distinctive feature of the approach proposed in [9] is that we do not introduce such a fluid by hand, but deduce its equations of motion from the requirement of the covariant divergenceless of the energy-momentum tensor (for other similar approaches see, for example, [47–50]). One of the purposes of the present paper was to show that the vector field defining such a fluid is already contained in the formalism of quantum field theory on a curved background. The existence of this vector (dynamical or not) allows one to define formally the gravitational and inertial forces as the 4-vectors (see [24, 30] and Eqs. (A5), (A6) below). Notice that for certain Euclidian approaches (see, e.g., [51–53]), where the Killing vector does not appear, the induced effective action contains the terms of the form $R\nabla^{-2}R$ which are essentially nonlocal.

These terms involve the inverse powers of the covariant d'Alambertian and the correct definition of such constructions requires the assignment of the initial and boundary conditions. In fact, this is equivalent to the introduction of the new fields into the model which also contribute to the covariant divergence of the energy-momentum tensor.

Observe also a remarkable feature of Eq. (99) – it is invariant with respect to dilatations of the Killing vector. This demonstrates the invariance of a theory with respect to a possible choice of the energy unit. If we introduce the world time associated with the Killing vector (when it is possible) then this variable should be also stretched in such a way that the product "energy"×"time" will be invariant. Therefore we shall call this symmetry as the energy-time symmetry. One may check that all the observables appearing in this paper are invariant under such a transform. However, if the theory possesses an intrinsic scale, say, the cutoff scale, this invariance is violated. Formally, the violation follows from the fact that the right-hand side of (99) must be regularized. As a result, the additional terms involving g_μ appear and break the symmetry. The typical terms violating the energy-time symmetry are presented in Eq. (21) of [9] (see also [54]). One can distinguish two types of such violating terms: hard violating and soft violating. The former scale as certain powers under dilations of the Killing vector, while the latter do it logarithmically. The coefficients at the soft violating terms are well known and nothing but the scaling functions (β or γ) of the renormalization group taken at that scale where the model is in a perturbative regime (if it exists). Note that all the particles of the model, even with large masses, contribute to this coefficients on equal footing (for a concrete calculations in the standard model see [55]). The relation of these coefficients with the scaling functions is not surprising (see, e.g., [10, 11]) as long as the energy-time symmetry involves a dilatation. Hence, if the theory were conformal on a quantum level, such anomalous terms would not arise. The restriction on the parameters of the models implying $\beta = 0$ can be found, for example, in [56]. According to [9], the anomaly of the effective action under the energy-time dilatations defines the enthalpy density of the condensate g_μ, although the hydrodynamic equations for g_μ are not empty even in the case of a vanishing enthalpy.

Appendix A: $(3+1)$ with the Killing vector

In this appendix, we collect some formulas regarding the differential calculus on the Riemannian manifold with the Killing vector. If the metric possesses the Killing vector ξ^μ then the following

useful relations hold

$$
\begin{aligned}
f_{\mu\lambda}g^{\lambda} = 0, \qquad g_{\lambda}h^{\lambda} = 0, \qquad g^{\lambda}\nabla_{\lambda}f_{\sigma\mu} = g_{[\mu}f_{\sigma]\lambda}h^{\lambda}, \\
g^{\mu}g^{\nu}\nabla_{\mu}h_{\nu} = g^2h^2, \qquad \nabla^{\lambda}f_{\lambda\mu}g^{\mu} = -\frac{1}{2}f^2, \qquad \nabla_{\mu}g_{\nu} = \frac{1}{2}f_{\mu\nu} - h_{(\mu}g_{\nu)},
\end{aligned} \tag{A1}
$$

and for the curvature

$$
\begin{aligned}
g^{\lambda}R_{\lambda\nu\sigma\mu} = \frac{1}{2}\nabla_{\nu}f_{\sigma\mu} - \frac{1}{2}h_{[\sigma}f_{\mu]\nu} + h_{[\sigma}g_{\mu]}h_{\nu} + f_{\sigma\mu}h_{\nu} - g_{[\sigma}\nabla_{\mu]}h_{\nu}, \\
R_{\mu\nu}g^{\mu}g^{\nu} = \frac{1}{4}f^2 - g^2\nabla_{\lambda}h^{\lambda}, \qquad g^{\lambda}R_{\lambda\mu} = f_{\mu\lambda}h^{\lambda} - \frac{1}{2}\nabla^{\lambda}f_{\lambda\mu} - g_{\mu}\nabla_{\lambda}h^{\lambda},
\end{aligned} \tag{A2}
$$

where $f_{\mu\nu} = \partial_{[\mu}g_{\nu]}$ and $h_{\mu} = \partial_{\mu}\ln\sqrt{|\xi^2|}$.

Using the Killing vector we can construct the projected connection,

$$
\bar{\Gamma}^{\alpha}_{\mu\beta} := \Gamma^{\alpha}_{\mu\beta} + \frac{\xi^2}{2}(f^{\alpha}{}_{(\mu}g_{\beta)} + 2h^{\alpha}g_{\mu}g_{\beta} - 2g^{\alpha}h_{(\mu}g_{\beta)}), \tag{A3}
$$

such that

$$
\begin{aligned}
\bar{\nabla}_{\mu}\bar{g}_{\rho\sigma} = 0, \qquad \bar{\nabla}_{\mu}\xi^{\nu} = 0, \qquad \bar{\nabla}_{\mu}g_{\nu} = \frac{1}{2}f_{\mu\nu}, \\
\bar{\nabla}_{\mu}g_{\rho\sigma} = \frac{\xi^2}{2}f_{\mu(\rho}g_{\sigma)} + 2\xi^2h_{\mu}g_{\rho}g_{\sigma}, \qquad \bar{\nabla}_{\mu}\bar{g}^{\rho\sigma} = -\frac{\xi^2}{2}f_{\mu}{}^{(\rho}g^{\sigma)}, \qquad \bar{\nabla}_{\rho}\bar{g}^{\rho\sigma} = 0.
\end{aligned} \tag{A4}
$$

Then the equation for geodesics of the metric $g_{\mu\nu}$ takes the form (cf. [24])

$$
\begin{aligned}
\nabla_u u^{\mu} = \bar{\nabla}_u u^{\mu} - (\xi u)(f^{\mu}{}_{\nu}u^{\nu} + (gu)h^{\mu} - 2(hu)g^{\mu}) = 0 \;\Rightarrow \\
\bar{g}_{\mu\nu}\nabla_u u^{\nu} = \bar{\nabla}_u(\bar{g}_{\mu\nu}u^{\nu}) - (\xi u)(f_{\mu\nu}u^{\nu} + (gu)h_{\mu}) = 0,
\end{aligned} \tag{A5}
$$

The second term on the left-hand side of the first equation can be regarded as the acceleration caused by a stationary gravitational field [30]. The first term in parentheses is associated with the so-called gravimagnetic force, whereas the second term is responsible for the gravielectric component of the force. The factor at these parentheses is the energy of the particle divided by its mass. In particular, the vector field ξ^{μ} allows one to define formally the inertial forces as the 4-vector

$$
f^{\mu}_{iner} := -m[\nabla_u u^{\mu} + (\xi u)(f^{\mu}{}_{\nu}u^{\nu} + (gu)h^{\mu}) - 2(hu)g^{\mu}] \;\Leftrightarrow\; f^{\mu}_{iner} + f^{\mu}_{g} + f^{\mu} = 0, \tag{A6}
$$

where $f^{\mu} := m\nabla_u u^{\mu}$ and $f^{\mu}_{g} := m(\xi u)(f^{\mu}{}_{\nu}u^{\nu} + (gu)h^{\mu} - 2(hu)g^{\mu})$ are the external (nongravitational) and the gravitational forces, respectively.

The Riemann and Ricci curvatures of the projected connection read as (see also [28])

$$\bar{R}^{\rho}{}_{\nu\sigma\mu} = R^{\rho}{}_{\nu\sigma\mu} - \frac{\xi^2}{2}\Big(f_{\sigma\mu}f_{\nu}{}^{\rho} - g_{[\sigma}\nabla_{\mu]}f_{\nu}{}^{\rho} - g_{\nu}\nabla^{\rho}f_{\sigma\mu} - \frac{1}{2}f_{\nu[\sigma}f_{\mu]}^{\ \ \rho}\Big) -$$
$$+ \xi^2\Big(g_{[\sigma}h_{\mu]}h_{\nu}g^{\rho} - g_{[\sigma}h_{\mu]}h^{\rho}g_{\nu} + \frac{1}{2}h_{[\sigma}f_{\mu]\nu}g^{\rho} - \frac{1}{2}g_{[\sigma}f_{\mu]\nu}h^{\rho} + g_{[\sigma}\nabla_{\mu]}h_{\nu}g^{\rho} - g_{[\sigma}\nabla_{\mu]}h^{\rho}g_{\nu} -$$
$$- f_{\sigma\mu}(h_{\nu}g^{\rho} - h^{\rho}g_{\nu}) - f_{\nu}{}^{\rho}h_{[\sigma}g_{\mu]} - \frac{1}{2}h_{[\sigma}f_{\mu]}^{\ \ \rho}g_{\nu} + \frac{1}{2}g_{[\sigma}f_{\mu]}^{\ \ \rho}h_{\nu}\Big) + \frac{\xi^4}{4}g_{\nu}g_{[\sigma}f_{\mu]}^{\ \ \lambda}(f_{\lambda}^{\ \ \rho} + 2h_{\lambda}g^{\rho}),$$
$$\bar{R}_{\mu\nu} = R_{\mu\nu} + h_{\mu}h_{\nu} + \nabla_{\mu}h_{\nu} - \frac{\xi^2}{2}[f_{\mu\rho}f^{\rho}_{\ \ \nu} + g_{(\mu}\nabla_{\rho}f_{\nu)}^{\ \ \rho} + 2g_{\mu}g_{\nu}(h^2 - \nabla_{\rho}h^{\rho}) + g_{(\mu}f_{\nu)}^{\ \ \rho}h_{\rho}] + \frac{\xi^4}{4}g_{\mu}g_{\nu}f^2, \tag{A7}$$

where $f^2 := f_{\mu\nu}f^{\mu\nu}$. Such a notation is used for scalars. As for matrices, $f^2 := f_{\mu\lambda}f^{\lambda}_{\ \ \nu}$ and then $\mathrm{Sp}\, f^2 = -f^2$. The scalar curvature becomes

$$\bar{R} = \bar{g}^{\mu\nu}\bar{R}_{\mu\nu} = R + 2\nabla_{\rho}h^{\rho} + \frac{\xi^2}{4}f^2. \tag{A8}$$

Besides, we have

$$\bar{\nabla}^i h_i = \bar{g}^{\mu\nu}\bar{\nabla}_{\mu}h_{\nu} = \nabla_{\lambda}h^{\lambda} - h^2, \qquad h_i h^i = h^2, \qquad f_{ij}f^{ij} = f^2. \tag{A9}$$

Also we need the quantities in the adapted coordinates in the weak field limit [24]

$$\bar{R}_{iklm} \approx \frac{r_g}{2r^3}(2\delta_{i[l}\delta_{m]k} - 3n_i n_{[l}\delta_{m]k} + 3n_k n_{[l}\delta_{m]i}), \qquad \bar{R}_{km} \approx \frac{r_g}{2r^3}(\delta_{km} - 3n_k n_m),$$
$$\bar{R}_{ik}\bar{R}^k_j \approx \frac{r_g^2}{4r^6}(\delta_{ij} + 3n_i n_j), \qquad \bar{R}_{iklm}\bar{R}_j^{\ klm} \approx \frac{r_g^2}{2r^6}(5\delta_{ij} - 3n_i n_j),$$
$$\bar{\nabla}^2\bar{R}_{ij}dx^i dx^j = \frac{3r_g^2}{2r^6}(1 - \frac{r_g}{r})^{-1}dr^2 - \frac{3r_g^2}{4r^6}(r^2 d\theta^2 + r^2\sin^2\theta d\varphi^2) \approx -\frac{3r_g^2}{4r^6}(\delta_{ij} - 3n_i n_j)dx^i dx^j, \tag{A10}$$

where $n_i = x_i/r$ and $\bar{\nabla}\bar{R}_{ij}$ is calculated for the Schwarzschild metric. Recall that $\bar{R} = 0$ and $\nabla^{\lambda}h_{\lambda} = 0$ for this metric.

Appendix B: Some expansions

In this appendix we give the expansions of some quantities appearing in the main text. For the metric in the Riemann normal coordinates we have [27, 29]

$$\bar{g}_{ij} = \delta_{ij} - \frac{1}{3}\bar{R}_{ikjl}y^k y^l - \frac{1}{3!}\bar{\nabla}_s\bar{R}_{ikjl}y^k y^l y^s + \frac{1}{5!}(\frac{16}{3}\bar{R}_{sjn}{}^{m}\bar{R}_{likm} - 6\bar{\nabla}_{sn}\bar{R}_{ikjl})y^k y^l y^s y^n + O(y^5),$$
$$-\bar{g} = 1 - \frac{1}{3}\bar{R}_{ij}y^i y^j - \frac{1}{3!}\bar{\nabla}_i\bar{R}_{jk}y^i y^j y^k - \frac{1}{4!}(\frac{6}{5}\bar{\nabla}_{ij}\bar{R}_{kl} + \frac{4}{15}\bar{R}_{mijn}\bar{R}^{m\ \ n}_{\ kl} - \frac{4}{3}\bar{R}_{ij}\bar{R}_{kl})y^i y^j y^k y^l + O(y^5). \tag{B1}$$

Also we deduce

$$|\xi^2|^{-1/4}(\mathbf{x})|\xi^2|^{-1/4}(\mathbf{y}) = |\xi^2|^{-1/2}(p)(1 - \frac{1}{8}\bar{\nabla}_i h_j\bar{\sigma}^i\bar{\sigma}^j + O(\bar{\sigma}^3)),$$
$$\bar{\Delta}^{-1}(\mathbf{x},\mathbf{y}) = (1 - \frac{1}{6}\bar{R}_{ij}\bar{\sigma}^i\bar{\sigma}^j + O(\bar{\sigma}^3)). \tag{B2}$$

In order to prove the relation (24), the following expansions may be useful

$$
\begin{aligned}
-1 &\equiv \xi^2(u^0)^2 + u_iu^i + 2\big[\xi^2((u^0)^2h_iu^i + u^0\dot{u}^0) + u_i\dot{u}^i\big]\tau + \\
&+ \Big\{\xi^2\big[(u^0)^2(h_i\dot{u}^i + 2(h_iu^i)^2 + \bar{\nabla}_ih_ju^iu^j) + 4u^0\dot{u}^0h_iu^i + (\dot{u}^0)^2 + u^0\ddot{u}^0 + \\
&+ \frac{u^0}{2}f_{ij}u^i\dot{u}^j\big] + (\dot{u}^i)^2 + u_i\ddot{u}^i\Big\}\tau^2 + O(\tau^3), \\
(\nabla_uu)^2 &= \xi^2(\dot{u}^0)^2 + (\dot{u}^i)^2 + \xi^2\Big[\xi^2(u^0)^2\big(h^2(u^0)^2 + 2u^0h^if_{ij}u^j - u^if^2_{ij}u^j\big) - \\
&- 2u^0(\dot{u}^if_{ij}u^j + u^0h_i\dot{u}^i) + 4h_iu^iu^0\dot{u}^0 + 4(h_iu^i)^2(u^0)^2\Big] + O(\tau), \\
\xi^2(p) &= \xi^2\Big[1 + h_iu^i\tau + \big(h_i\dot{u}^i + (h_iu^i)^2 + \frac{1}{2}\bar{\nabla}_ih_ju^iu^j\big)\frac{\tau^2}{2}\Big] + O(\tau^3).
\end{aligned}
\tag{B3}
$$

Here all the fields are taken at the origin of the system of coordinates described after Eq. (21). Notice that $u^0 = g_\mu u^\mu$ in this frame.

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